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AN APPROACH TO SYSTEM PERFORMANCE PREDICTION

BY

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ABSTRACT

Part I reviews some past developments toward a sound basis for "reliability" as an evaluation factor in combination with "inherent accuracy." The method of zero failure probability is described and evaluated. An approach by Brown is discussed and an alternative method related to it is suggested, with brief comments on some practical problems of implementing this approach.

In Part II, analytical aspects of a method for describing system performance in terms of both reliability and accuracy are treated. System value and value measures are discussed and a generalized probable error is chosen as a measure of system performance. Some general characteristics of the solution for this value measure are found, with emphasis on establishing upper and lower bounds independent of the precise forms of the accuracy and reliability distribution functions. The concepts of "failed error" and "effective reliability" are introduced and elaborated. Some numerical results are given for the special case of circular normal distribution functions.

Part I. Reliability as an Evaluation Factor*

1. BACKGROUND OF THE PROBLEM

The degree to which a system accomplishes its basic task often can be measured by a single characteristic we call accuracy. For such systems design criteria, quantitative performance specifications, and the evaluation of the system worth traditionally have been based on inherent[2] accuracy as a fundamental system property.

More recently it has become generally recognized that there are many other system characteristics of considerable operational importance. Some of these bear on the system's operational utility for reasons quite divorced from accuracy—from considerations of logistics, economics, operating procedure, or interaction with other systems, for

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[2] That is, accuracy achieved when all parts of the system operate within their original design tolerances.

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example. Other characteristics are significant not only because of such peripheral considerations, but also because they, together with inherent accuracy, affect the net or "effective" accuracy of the system.

Reliability, the probability of "non-failure" of the system under specified circumstances, is one of the latter class which has been paid particular attention in the last few years. I want to stress the importance of remembering that the reliability concept actually enters both sets of considerations mentioned above, and that its roles in relation to accuracy and to logistics, etc. are essentially different. These roles are sometimes confused because increasing reliability, in itself and other things being equal, is desirable in both cases. However, they should be distinguished carefully because they may involve different reliability goals, different kinds of operations with the system "reliability number," different practical consequences of reliability changes, different problems of reliability measurement, and even different interpretations of the word "reliability."

Reliability is considered here only as an evaluation factor entering the effective accuracy of the system. "Effective accuracy," for present purposes, is equated to "system performance," implying that other factors contributing to operational value are ignored. There is some danger in this, of course, if it is forgotten that the optimization of system performance in this sense is actually just a sub-optimization, with all the traps inherent in sub-optimizations (3).[3] Nevertheless, focusing on effective accuracy alone is justified in the many cases of practical interest where it really is the dominant factor in determining system value, provided all the important ingredients of effective accuracy are properly included.

This paper attempts to develop a rational method of determining system performance embracing both inherent accuracy and reliability concepts. Part I briefly reviews some of the past developments toward a sound basis for "reliability" as an evaluation factor. The method of zero failure probability is discussed first, with its advantages, disadvantages, and modifications. Next, an "ultimate" method is described, together with an approach to it by Brown. Finally, an alternative approach is proposed and compared in some detail with Brown's. Part I concludes with a summary of the problems which must be solved to implement the method and the status of these problems.

2. METHOD OF ZERO FAILURE PROBABILITY

"System reliability" has a well known and widely accepted definition in terms of the probability that there be no system failures (suitably qualified as to circumstances). It is equally well known that a major difficulty in applying this usual definition is the problem of deciding what constitutes satisfactory performance of the system (5). In other words, what is a system failure?

[3] The boldface numbers in parentheses refer to the references appended to this paper.

One approach, exemplified particularly by the work of Lusser (7), equates "system failure" to "failure of any component." This can be described as a "zero-failure probability" measure of reliability. It shifts the burden of specifying what a failure is from the over-all system to its component parts, whose behavior is presumed simple enough that the meaning of failure is unambiguous.

There are some advantages to treating failures in this way. It is a method simple in concept and application, permitting a relatively easy calculation of system reliability from the failure distributions of the parts. One is free of concern about the time propagation effects of component failure on system performance. Reliability numbers so calculated may be useful as indices for motivating and monitoring reliability improvement efforts.

However, this zero (component) failure probability has a serious and fundamental defect as a measure of reliability. It is inadequate to describe the behavior of systems in which "failure" is not a catastrophic event, but rather a gradual degradation of some other performance factor, such as accuracy. There is no obvious bridge between this reliability value and the over-all system effectiveness or operational worth; in general, there is no way to apply it to problems of rational assessment of the importance of unreliability relative to other evaluation factors. It also has limitations as a reliability improvement index, for it weighs equally equal improvements in any two components, even though one component may completely dominate the system performance. Applied on the component level, zero-failure probability can result in a gross over-statement of the operational importance of failures.

It has been generally recognized for some time that these difficulties exist, that failures *within* many systems are not identical with or equivalent to failures *of* the system: moreover, that one usually is not interested in internal failures *per se*, but in the net system performance with all factors considered. Carhart (2), for example, has given an excellent discussion of this question, pointing out that the unequal operational importance of various failure modes is very troublesome in defining system reliability. Knight and Jervis (4) state that "the invalidity of the assumption that the failure of any component causes the system to fail is readily seen from a fact disclosed by ARINC findings. . . ." Recognition of the deficiency of the zero-failure probability approach is explicit and not infrequent in the proceedings of a recent reliability symposium.[4]

There are various ways in which one can try to surmount the problem. One is effectively to lump the system into a set of series and parallel "elements"[5] and to equate system failure to the failure of a series element. Still based on a zero-failure probability, the method now

[4] Joint Military-Industry Guided Missile Reliability Symposium, 15–17 October 1956, Redstone Arsenal, Huntsville, Ala.

[5] Which may be basic components, subsystems, or any necessary mixtures of these.

permits some failures to be ignored in calculating reliability. It retains the advantages described previously but would be expected to reflect operational importance better. However, it generally is hard to implement in particular cases because of the difficulty of deciding how to perform the decomposition into series and parallel elements. Performing the decomposition is equivalent to making a semi-subjective distinction between "significant failures," which are to be counted, and "insignificant failures" which are to be ignored.

The fact that this approach still involves a two-valued decision—either an element failure causes a "complete" system failure or none at all—means that it is still but a half-way point. One must go further to make provision for the general case of gradual performance degradation. Nevertheless, it seems to be an important improvement over the previous zero-failure probability approach counting all components; and, in fact, may be as advanced a method as is practical to apply to many systems. A technique equivalent to this is now being used on an investigative basis by IBM (6).

The preceding comments deal primarily with methods of deriving a "system reliability" number. After the number is obtained, what is done with it as an evaluation factor? The convention has been that all failed systems are complete losses. Therefore, the reliability number has one application as a measure of wastage, with all the operational implications of wasted systems. However, it does also change the effective accuracy. If all failed systems are assumed to have infinite error, the probable error (or similar measure) for the family of all systems, failed and unfailed, is increased over that of the family of unfailed systems.

This is a fair use of the number, but the difficulty still exists that not *all* failures lead to infinite error. To assume they do *may be* to understate the system effectiveness or value. On the other hand, the degree of understatement of value may be insignificant and ignorable. But if this is true, it must be demonstrated within the framework of a more general point of view rather than merely assumed. To recognize the distinction between "severe" and "mild" failures and to evaluate the degree to which one can ignore this distinction is a basic goal of the more general approaches discussed below.

3. BROWN'S METHOD

What might be termed the ultimate approach, one which is probably most satisfying on logical grounds and at the same time most difficult to put into practice, is to try to estimate quantitatively the operational importance and probability of occurrence of each failure mode. These factors then can be used to attempt a direct calculation of the operational effect of failures. This is the method Carhart has called the "performance spread" method, and whose feasibility he seriously questioned.

An application of such a method to a simple circuit, which has all the elements of the complete sophisticated approach, has been described by Brown (1). The characteristics of his approach are as follows:[6]

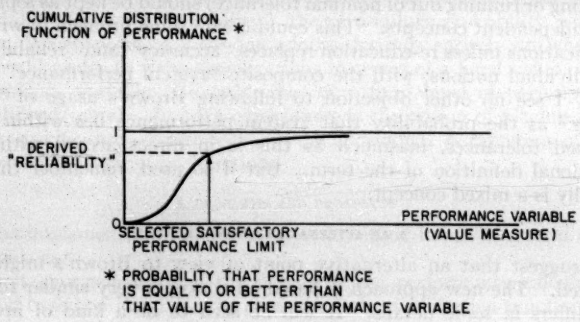

CUMULATIVE DISTRIBUTION FUNCTION OF PERFORMANCE *
1
DERIVED "RELIABILITY"
0
SELECTED SATISFACTORY PERFORMANCE LIMIT
PERFORMANCE VARIABLE (VALUE MEASURE)
* PROBABILITY THAT PERFORMANCE IS EQUAL TO OR BETTER THAN THAT VALUE OF THE PERFORMANCE VARIABLE

FIG. 1. Brown's method of reliability prediction.

1. a performance variable (in his case, circuit gain) is defined and used as a value measure;
2. a performance distribution curve is obtained as the probability density of having the performance variable at various levels;[7]
3. a value of the performance variable is selected as a satisfactory performance limit; and
4. the corresponding probability is read from the *cumulative* distribution curve and called the system "reliability."[8]

It is the derivation of this probability of satisfactory performance, the prediction of system reliability, which is his goal.

An important part of his work concerns the derivation of the performance distribution function. The performance change of a physical circuit was empirically, linearly, related to deliberately induced variations of component performance. The means and variances of component property distributions were then used to predict the corresponding statistical characteristics of circuit performance.

This approach is a sound one, but I believe that some difficulty can arise on one point. Specifically, the reliability defined in this method is not uniquely related to failures within the system. By including all factors which influence net system performance, it also embraces the

[6] These steps are illustrated graphically in Fig. 1.

[7] This might equally well be described by a cumulative performance distribution function, and will be so described in what follows.

[8] Strictly, this is done after the performance distribution is multiplied by a weighting function which assesses the "value" of each level of performance. This is equivalent to transforming the performance variable suitably so that the scale of the horizontal axis of Fig. 1 directly measures "value," while leaving the distribution function alone.

concept of inherent accuracy. While there is nothing really wrong with this, it sacrifices the convention of long standing that system accuracy (with everything "working properly") and of reliability (as something breaking or running out of nominal tolerance) should be kept as separate and independent concepts. This could create some problem in writing specifications unless re-education replaces "accuracy" and "reliability," as individual notions, with the composite "system performance" concept. I see no other objection to following Brown's usage of "reliability" as the probability that system performance lies within preassigned tolerances, inasmuch as this is in direct accord with the traditional definition of the term. But if so used, remember that it actually is a mixed concept.

4. AN ALTERNATIVE APPROACH

I suggest that an alternative point of view to Brown's might be adopted. The new approach is based on concepts very similar to his, but differs in some details. It will be seen to be a kind of inverse method to Brown's, one which is more or less preferable according to point of view adopted in establishing the system specifications.

The characteristics of the proposed approach are as follows:[9]

1. a performance variable (in this case, probable error or a generalization of it) is defined and used as a value measure;
2. a family of performance distribution curves is obtained, each representing a cumulative distribution function of the performance variable;
3. a probability level for the performance variable is selected; and
4. the corresponding value of the performance variable is read from one of the cumulative distribution curves.

It is the derivation of system performance at a given probability level which is the goal of this approach—a sort of inverse to Brown's.

The second difference arises from the fact that a family of distribution functions is considered, whose characteristic parameters are the statistical parameters of the inherent accuracy as conventionally defined, the system "reliability" as defined in the zero-failure probability approach described previously, and a set of new parameters which may be called "failure severity" parameters and which measure the statistical effect of a failure on system effective accuracy when the failure does occur. Therefore, the conventional ideas of independent accuracy and reliability can be retained, provided the new concept of failure severity is adjoined. By setting the derived performance variable at a specified value, one obtains a functional relationship among these parameters, a "trade-off" relationship which can be used to assess the relative importance of parameter changes.

[9] These steps are illustrated graphically in Fig. 2.

I hasten to state that Brown's approach, with performance variable fixed and performance probability derived, can be applied equally well using this same family of curves. The only precaution which is necessary is to distinguish carefully his "reliability," the derived performance probability, from my "reliability," the zero-failure probability parameter.[10] Which point of view one adopts depends entirely on whether one would prefer to write specifications in terms of a fixed value of the performance variable (Brown's case) or a fixed level of performance probability (mine). In both cases, there is the option of placing additional specification constraints on the individual parameters of the performance distribution function.[11]

5. PROBLEMS AND PROSPECTS

The implementation of the proposed method involves several steps or problems.

First, one can examine the details of the performance calculation method based on Fig. 2 for the case where the distribution functions for

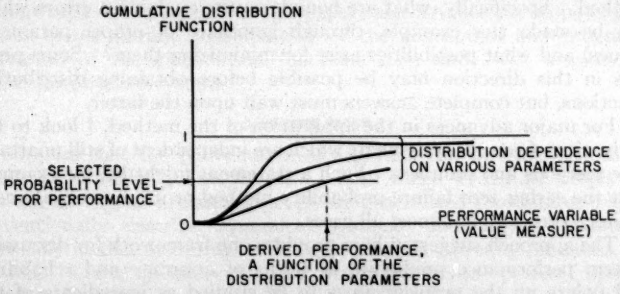


FIG. 2. Characteristics of the present approach.

inherent accuracy and failed error can be assumed given. It is possible to make some general remarks about the nature of the solution and to establish some upper and lower bounds on it which are independent of the detailed forms of these distributions. This topic will be treated in Part II of this paper.

Second, one must find ways—first conceptual, then practical—for deducing the forms of the required distribution functions. These problems undoubtedly are among the most difficult in the practical implementation of the method. Various possibilities exist. Brown,

[10] Admirably illustrating one of the basic difficulties in many current reliability applications.

[11] I am in full accord, incidentally, with Brown's implication that the truly proper place to impose a specification is on the *over-all* performance, not on the individual parameters *exclusively*.

for example, found the performance distribution function empirically as described in Section 3. A more basic and admittedly much more difficult analysis is to derive the system distribution functions from the distribution functions of the components. This approach can be very successful in finding the inherent accuracy distribution function,[12] although the more sophisticated ways of doing this do not seem to be widely known. However, the complete analog does not now exist for systems which fail (*a fortiori* when the failure changes the system topology). Significant advances are required in this area, but a practical solution does not seem unattainable.

Third, knowledge of the forms of the distribution functions must be supplemented by a practical method for finding numerical values of the distribution parameters. This involves the whole mass of problems of reliability testing and inference from systematic failure reports. Although the analogous information regarding accuracy is much easier to obtain, it is clear that the distribution parameter values are the center of a major problem area.

A fourth and final general problem is that of error sources in the method. Specifically, what are bounds on the evaluation errors which can be made (for example, through ignorance of proper parameter values) and what possibilities exist for minimizing them? Some progress in this direction may be possible before obtaining distribution functions, but complete answers must wait upon the latter.

For major advances in the application of the method, I look to the derivation of general statements which are independent of still unattainable methods and numbers. Such a statement might be, for example, that the earlier zero failure probability method or its generalizations is entirely adequate in almost all cases.

The approach suggested here provides one framework for discussing system performance prediction in terms of accuracy and reliability, and points up the problem areas to be studied as ingredients of the general solution. In its subsequent elaboration, I believe that the approach also will furnish considerable insight into the nature of this solution. Therefore, the point of view presented here may be of some value even if not all of the problems outlined above can be solved completely.

Part II. Analytical Foundations

6. INTRODUCTION

In Part I, I described qualitatively a possible approach to system performance prediction using reliability and inherent accuracy as evaluation factors. As presented there, the method applies to systems whose operational value can be measured by a single performance variable, effective accuracy. An important step in implementing the method is

[12] This also seems capable of generalization to encompass some types of failure.

to bring forth its analytical foundation and to use this basis to deduce as much as possible about the general nature of the solution.

This is the problem area of Part II. First, notation and assumptions about system value in terms of a performance number are made explicit. Second, an implicit expression is developed for performance in terms of distribution functions related to reliability and inherent accuracy. Third, the nature of the solution of this equation for the performance number is examined insofar as possible without recourse to explicit distribution functions. Fourth, an example is given using two dimensional (circular) normal distributions for both evaluation factors. Finally, a concept of effective reliability is introduced and discussed, illustrated by the case of circular normal distribution functions.

It is shown how the system performance, as measured by a generalized probable error, depends on the reliability (in the zero-failure probability sense), accuracy parameters, and another class of parameters it is convenient to call "failure-severity parameters." It is clear that with such a relationship it is possible to assign a performance number and to examine trade-offs among the various parameters. This important topic is not treated in this paper; neither does the scope of the present paper permit consideration of the conceptual and practical derivation of the distribution functions $A(\epsilon)$ and $F(\epsilon)$, introduced below, for physical systems.

7. SYSTEM VALUE

Suppose that the performance of a system can be described completely by N scalar quantities $(x_1, \cdots, x_N)$ which we denote simply as a vector $\mathbf{x}$. These are analogous to the generalized coordinates which conventionally describe dynamical systems, and may be realized physically as dial or counter readings, voltages, currents, pressures, or the like. Let $\mathbf{x}^*$ be the value of $\mathbf{x}$ which represents "perfect" operational performance.

Define a scalar "system error" ϵ by $\epsilon = \text{norm}\ (\mathbf{x} - \mathbf{x}^*) \equiv \|\mathbf{x} - \mathbf{x}^*\|$. The nature of the norm must be specified for the particular system and its application under consideration. As one example, visualize the case of bombardment where $\mathbf{x}$ is the vector hit position of the bomb (an output of the over-all system consisting of the bomb-carrier, its navigation and bombing systems, and the atmosphere through which the bomb falls). If this over-all system is operating perfectly without (uncorrected) built-in systematic errors,[13] $\mathbf{x}^*$ is the vector target position. A conventional performance factor is the system error as reflected by radial miss distance. Denoting this radial miss distance by ϵ, the appropriate norm is simply Euclidian distance, and $\epsilon = \sqrt{|\mathbf{x} - \mathbf{x}^*|}$.

[13] Systematic errors are often permitted in practice if the mechanization to remove them results in a performance gain which is small out of proportion to the increased complexity, weight, cost or similar factors.

As a more sophisticated example, consider the problem of locating the coordinates of ground points by a reconnaissance system. If ϵ denotes the location error, three components of $\mathbf{x} - \mathbf{x}^*$ are the three position coordinate errors of the system; and three additional components of $\mathbf{x} - \mathbf{x}^*$ are errors in the system's three orientation angles. The geometry relating ϵ to the $\mathbf{x} - \mathbf{x}^*$ components requires a norm of the form $\|\mathbf{x} - \mathbf{x}^*\| = \sqrt{\sum \sum a_{ij}(\mathbf{x}_i - \mathbf{x}_i^*)(\mathbf{x}_j - \mathbf{x}_j^*)}$. That is, ϵ^2 is a general quadratic form in the $\mathbf{x} - \mathbf{x}^*$ components.

The present treatment is directed toward systems whose operational worth is measured by the single characteristic ϵ. That is, there exists a value function $V(\epsilon)$ which is non-increasing with ϵ. In almost every case, of course, any reasonable estimate of value must encompass a number of other evaluation factors, but these are often intangible or at least (presently) non-quantitative system characteristics. For a large class of systems, it is not unreasonable to use ϵ as a single quantitative evaluation factor, provided that all effects contributing to it are included properly.

Two measures of value, that is, two value functions, are of particular interest. One of these is the expected value $\mathfrak{E}(V)$. Denote by $P(V_0)$ the probability that $V \geq V_0 \equiv V(\epsilon_0)$. Then the expected value is given by

$$\begin{aligned} \mathfrak{E}(V) &= \int_0^\infty P(V_0)dV_0 = \int_\infty^0 pr\{\epsilon \leq \epsilon_0\} \frac{dV(\epsilon_0)}{d\epsilon_0} d\epsilon_0 \\ &= \int_0^\infty p(\epsilon_0) V(\epsilon_0) d\epsilon_0 \end{aligned} \tag{1}$$

where $p(\epsilon_0)$ is the probability density function for the variable ϵ.

A case of practical importance where this integral can be evaluated directly is that to which the lethal "radius"[14] concept applies.[15] That is, V equals a non-zero constant for ϵ less than a specified value and zero for ϵ greater than that value. This requires, of course, that $\epsilon = 0$ be the center of the lethal hypervolume, which will be true when the latter is centered at $\mathbf{x} = \mathbf{x}^*$. In general in this kind of application, V might be considered a sort of damage or kill probability function.

Another measure I choose to call the "p-error," ϵ_p, a generalization of the probable error often used as a performance number for physical systems. The p-error is defined as the solution of the equation

$$pr\{\epsilon < \epsilon_p\} = p \tag{2}$$

[14] In our case, actually a lethal hypersphere, hypercube, or some other such region depending on the choice of norm.

[15] The nomenclature is especially appropriate to guided weapons, but the situation is the same whenever the system has two states: worthless and satisfactory.

with p an assigned number in the range $(0, 1)$. If $p = 1/2$, for example, ϵ_p is the system probable error.

The present discussion is limited to ϵ_p as a value measure. Admittedly, this is not the best measure for many purposes but it is quite convenient for others. The probable error ($p = 1/2$), in particular, is a conventional measure of system performance which has wide application. In any case, it will be clear that many of the same considerations would apply to an analogous treatment of $\mathfrak{E}(V)$.

8. PERFORMANCE DETERMINATION

It is easy in principle to find ϵ_p from the accuracy and reliability characteristics of the system. First define

"reliability" R = prob {no component failure}

"inherent accuracy" $A(\epsilon_0)$ = prob {if no component failure, then $\epsilon \leq \epsilon_0$}

"failed accuracy" $F(\epsilon_0)$ = prob {if one or more component failures, then $\epsilon \leq \epsilon_0$}.

Here R can be interpreted as the zero-failure probability in the classical sense on the component or subsystem level. However, a formally identical treatment applies if only "significant elements" are counted. There is some advantage to the latter in reducing the number of failure modes which need to be considered in finding F. (This may be feasible, though, only if one can easily recognize and discard from considerations elements which either contribute a negligible error relative to that from inherent accuracy alone, or no error at all, as might be the case if the elements are included in the system for reasons other than accuracy.)

Both $A(\epsilon_0)$ and $F(\epsilon_0)$ also implicitly contain a number of parameters from the individual probability distributions of which they are compounded. Those of A are denoted by the vector $\boldsymbol{\alpha} = (\alpha_1, \cdots, \alpha_{K_1})$ and those of F by $\boldsymbol{\phi} = (\phi_1, \cdots, \phi_{K_2})$. These parameters, $\boldsymbol{\phi}$ a kind of "failure-severity parameter," are discussed further in connection with the derivation of A and F. When it is important that they appear explicitly, we write $A(\epsilon_0, \alpha)$ and $F(\epsilon_0, \boldsymbol{\phi})$.

Now $P(V_0)$ can be interpreted as the probability that $\epsilon \leq \epsilon_0$ where $V_0 = V(\epsilon_0)$. Thus Eq. 2 and the definitions at the beginning of this section imply that ϵ_p satisfies

$$p = RA(\epsilon_p) + (1 - R)F(\epsilon_p). \tag{3}$$

Remember that the solution ϵ_p of Eq. 3 measures the value or performance of the system under the assumptions of this paper. It is a function of the value of p, of course, which we can visualize as establishing a performance level. In addition, it depends on the reliability R, the accuracy parameters $\boldsymbol{\alpha}$ and the failure-severity parameters $\boldsymbol{\phi}$, as well as being a functional of A and F.

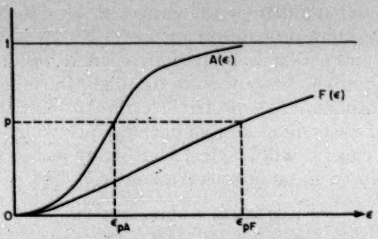


FIG. 3. Typical $A(\epsilon)$ and $F(\epsilon)$.

Supposing for the moment that $A(\epsilon)$ and $F(\epsilon)$ are given for all ϵ, how are these functions used to find the system performance as characterized by the ϵ_p-value? The solution is nicely visualized graphically. Consider the behavior of the functions

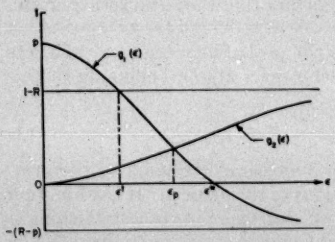
$$\left.\begin{aligned} g_1(\epsilon) &= p - RA(\epsilon) \\ g_2(\epsilon) &= (1-R)F(\epsilon) \end{aligned}\right\} \tag{4}$$

with ϵ. They intersect at an ϵ-value which equals ϵ_p by virtue of Eq. 3.

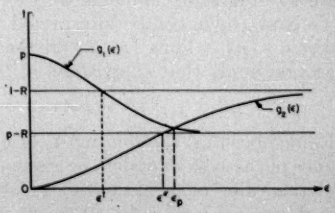
Now A and F, as cumulative distribution functions, both increase from zero to unity as ϵ goes from zero to infinity, and are non-decreasing. Typical forms are shown in Fig. 3. (While smooth curves of the type shown may be obtained in most applications, in principle both curves can have discontinuities characterized by a non-zero probability of quantized values of ϵ, and can have discontinuous derivatives.) Two useful quantities, ϵ_{pA} and ϵ_{pF}, are shown in the figure, defined by

$$A(\epsilon_{pA}) = p, \quad F(\epsilon_{pF}) = p. \tag{5}$$

Recall that p is typically, though not necessarily, equal to 1/2. It is



(A) Shown for $R > p$, $p > 1/2$.



(B) Shown for $R < p$, $p > 1/2$.

FIG. 4. Solution for the p-error.

convenient, and is not an important limitation, to suppose henceforth that $p \geq 1/2$.

Using the curves of Fig. 3 to find the functions $g_1(\epsilon)$ and $g_2(\epsilon)$ defined by Eq. 4, the curves of Fig. 4 result. Two cases are shown: (*A*) $R > p$ and (*B*) $R < p$, reflecting, respectively, negative and positive values of the g_1-asymptote. In both cases, p is shown exceeding 1/2, and in (*B*) it is further assumed for definiteness than $R > 1 - p$. However, the latter two conditions are in no way essential. The solution ϵ_p is indicated in both places, and certain auxiliary values of ϵ which are useful later below are noted in the figure.

9. SOME GENERAL PROPERTIES OF ϵ_p

Using no more than the qualitative description above of the graphical solution for ϵ_p, it is possible to make a number of statements about the solution properties.

First, it is clear that the following theorem results immediately from the non-decreasing character of A and F.

Theorem 1: Let ϵ_p be any solution of Eq. 3. If $A(\epsilon)$ and $F(\epsilon)$ are both constant in some maximal interval τ which includes ϵ_p, then all τ-values contained in τ are solutions of Eq. 3 but there are no solutions outside of τ. If there is no interval including ϵ_p within which $A(\epsilon)$ and $F(\epsilon)$ are both constant, then ϵ_p is a unique solution.

It is convenient to define solutions for certain discontinuous cases. Let ϵ_0 be a point of discontinuity of either $g_1(\epsilon)$ or $g_2(\epsilon)$ or both, with $g_1(\epsilon^-) \geq g_2(\epsilon^-)$ and $g_1(\epsilon^+) \leq g_2(\epsilon^+)$. Then ϵ_0 is defined to be a solution of Eq. 3. This enables us to state a second theorem which follows directly from the fact that $g_1(0) \geq g_2(0)$ and $g_2(\infty) \geq g_1(\infty)$.

Theorem 2: A solution ϵ_p of Eq. 3 always exists.

Now it does not seem to be a severe restriction in practical cases to assume that g_1 and g_2 have no common range within which they are constant and equal. With this assumption and in view of Theorems 1 and 2, we have

Theorem 3: There exists exactly one solution ϵ_p of Eq. 3.

The important difference between the two cases $R > p$ and $R \leq p$ is reflected by the next theorem. First define $\epsilon\dagger$, ϵ^*, and ϵ'' by

$$\left.\begin{array}{ll} A(\epsilon\dagger) = 1 - (1-p)/R & (1-p \leq R) \\ A(\epsilon^*) = p/R & (p < R) \\ F(\epsilon'') = 1 - (1-p)/(1-R) & (p > R) \end{array}\right\}. \tag{6}$$

Theorem 4: Assume $p \geq 1/2$. Then for $R > p$, $\epsilon\dagger \leq \epsilon_p \leq \epsilon^*$. For $1 - p \leq R \leq p$, $\max(\epsilon\dagger, \epsilon'') \leq \epsilon_p$. For $R < 1 - p$, $\epsilon'' \leq \epsilon_p$.

In other words, for $R > p$ one is able to bound the solution from above, but for $R \leq p$ this is not possible.

It is of some interest that ϵ_{pA} defined previously (Eq. 5) is in the same range as ϵ_p for $R > p$. That this is true is evident from Fig. 5.

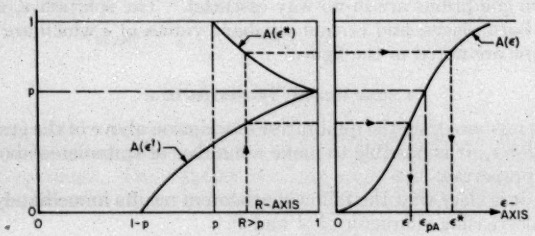


FIG. 5. Relations among $\epsilon\dagger$, ϵ^*, and ϵ_{pA}.

The left-hand side of that picture shows $A(\epsilon^*)$ and $A(\epsilon\dagger)$ given by Eq. 6 as a function of R for a fixed p; these functions intersect at the level p for $R = 1$. At a given value of $R > p$, the values of these two functions are picked off and projected onto the function $A(\epsilon)$, sketched on the right of the figure, to show $\epsilon\dagger$ and ϵ^*. Now ϵ_{pA} is just the value at which $A(\epsilon) = p$, which is seen to always satisfy $\epsilon\dagger \leq \epsilon_{pA} \leq \epsilon^*$. The same result, of course, can be obtained very easily from Eqs. 5 and 6 using the inequality $R > p$. However, the graphical point of view also shows very clearly how the solution range narrows on ϵ_{pA} as $R \to 1$. These results can be summarized as

Theorem 5: For $R > p$, $\epsilon\dagger \leq \epsilon_{pA} \leq \epsilon^*$. As $R \to 1$, $\epsilon^* - \epsilon\dagger \to 0$.

It is obvious that if $A(\epsilon)$ is very steep between the A-values $1-(1-p)/R$ and p/R, $\epsilon^* - \epsilon\dagger$ also is small.

One other simple bound is easy to establish. Consider Fig. 6 in which the case $R > p$ of Fig. 4 is repeated, now with the level $p(1 - R)$ added. From Eq. 4, $g_1(\epsilon)$ crosses that level when $A(\epsilon) = p$ and $g_2(\epsilon)$ crosses it when $F(\epsilon) = p$; that is, at ϵ_{pA} and $\epsilon_p 2$, respectively. It is apparent that the solution ϵ_p must lie between these values, regardless of whether $\epsilon_{pF} > \epsilon_{pA}$ (as shown in Fig. 6) or whether $\epsilon_{pF} > \epsilon_{pA}$. Now if $\epsilon_{pF} < \epsilon_{pA}$, it follows that $\epsilon_p < \epsilon_{pA}$, which is to say the system is better when failures occur than when they do not, and that $R = 0$ would be an optimum reliability. This is not the case for a properly designed system. Therefore, we impose the requirement on physical grounds that $\epsilon_{pF} \geq \epsilon_{pA}$. This being true, we know that $\epsilon_{pA} \leq \epsilon_p$. But $\epsilon\dagger \leq \epsilon_{pA}$,

so we have arrived at a greater lower bound than that quoted in Theorem 4. It can be stated as

Theorem 6: Assume $p \geq 1/2$. Then for $R > p$, $\epsilon_{pA} \leq \epsilon_p \leq \epsilon^*$.

Because of the general bounds on the solution that exist for $R > p$, it would be nice if we could restrict our considerations to this case. Unfortunately, this does not seem to be possible. To be sure, systems of low reliability generally are not considered operationally satisfactory, but sometimes they are tolerated.

The foregoing results about exhaust the elementary properties of the p-error obtainable without giving $A(\epsilon)$ and $F(\epsilon)$ more specific forms. Therefore, let us turn to a specific illustrative example.

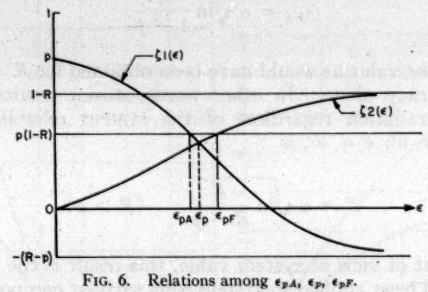


FIG. 6. Relations among ϵ_{pA}, ϵ_p, ϵ_{pF}.

10. CIRCULAR NORMAL A AND F

It is clear that the major problem in an actual case will be to determine the correct forms of A and F, based on a model of the system, and the values of the parameters which enter these distribution functions, based on the behavior of the system components. For illustration, we consider the specific simple case in which both A and F are circular normal. This case is not devoid of intrinsic interest, and is one for which some of the parameter variations can be studied easily.

Suppose that

$$\left.\begin{aligned} A(\epsilon) &= 1 - \exp(-\epsilon^2/\alpha^2) \\ F(\epsilon) &= 1 - \exp(-\epsilon^2/\phi^2) \end{aligned}\right\}. \tag{7}$$

The parameter α is the rms. circular error when no failures occur, therefore an inherent accuracy parameter. The parameter ϕ is the rms. circular error for the class of systems in which one or more component failures has occurred. Equation 3 becomes in this case

$$R\exp(-\epsilon_p^2/\alpha^2) + (1 - R)\exp(-\epsilon_p^2/\phi^2) = 1 - p. \tag{8}$$

It is natural to regard

$$\sigma = (\phi - \alpha)/\alpha \tag{9}$$

as a "failure-severity" parameter. We assume that in a properly designed system the accuracy cannot be statistically less in a failed than in an unfailed system, so that we must have $\phi \geq \alpha$ or $\sigma \geq 0$. Failures have zero severity if they do not broaden the error distribution at all. As ϕ becomes larger than α the failure severity increases, finally becoming infinite in the case that all failed systems have infinite error, thereby contributing nothing to the operational value of the system.

If $\sigma = 0$, the p-error has the specific value

$$\epsilon_{pA} = \alpha \sqrt{\ln \frac{1}{1-p}}. \tag{10}$$

This is the same value as would have been obtained for $R = 1$ from the inherent accuracy alone. In other words, $\phi = \alpha$ results in no performance degradation regardless of the system reliability number. Another result, for $\sigma = \infty$, is

$$\epsilon^* = \alpha \sqrt{\ln \frac{R}{R-p}} \qquad (R > p). \tag{11}$$

From the point of view of system value, this result is the same as if a unit value had been applied to system runs without component failure, and a zero value had been applied to failed systems. The same result for ϵ_p as given by Eq. 11 is obtained from Eq. 3 in which the $F(\epsilon_p)$ term is just arbitrarily omitted; that is, the dependence of effective system error on reliability is just ignored. Therefore, this case $\phi = \infty$ represents the situation encountered in the usual application of the zero-failure probability concept.

In its intermediate ranges, σ (or ϕ) becomes a new performance parameter to be adjoined to the customary accuracy (measured by α) and reliability (measured by R) to represent system behavior in cases where unreliability means simply a degradation in effective accuracy.

It is convenient to normalize ϵ_p to ϵ_{pA}. Define $\xi = \epsilon_p/\epsilon_{pA}$. Then Eq. 8 becomes

$$R(1-p)^{-1+\xi^2} + (1-R)(1-p)^{-1+\xi^2/(1+\sigma)^2} = 1. \tag{12}$$

Zero degradation from failure corresponds to $\xi = 1$; infinite degradation to $\xi = \infty$. This equation implicitly defines the function $\xi(R, \sigma)$ or the more basic $\epsilon_p(\alpha, R, \sigma)$. Although it generally cannot be solved explicitly for ξ, it is easy to find σ as an explicit function of ξ, R:

$$\sigma = -1 + \frac{\xi}{\sqrt{1 + \ln\left[\frac{1 - R(1-p)^{-1+\xi^2}}{1-R}\right] \Big/ \ln(1-p)}} \tag{31}$$

for $(1 \leq \xi < \infty, 0 \leq R \leq p)$ and $(0 \leq \xi < \xi^*, p < R \leq 1)$ where

$$\xi^* = \sqrt{\ln \frac{R}{R-p} \Big/ \ln \frac{1}{1-p}}. \tag{14}$$

The values of ξ^* are important because for $R > p$ they are asymptotes beyond which ξ cannot go regardless of how large σ grows. They are shown in Fig. 7 as a function of R for $p = 1/2$ (that is, ϵ_p is the

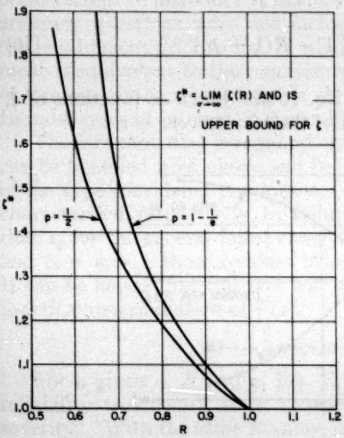


FIG. 7. ξ^* *versus* R for constant p.

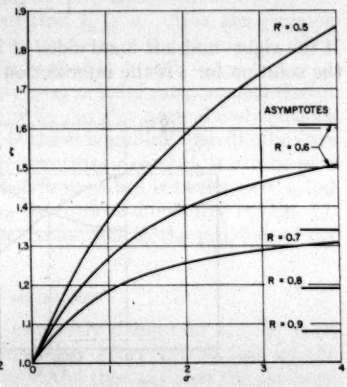


FIG. 8. ξ *versus* σ for constant R and $p = 1/2$.

probable error), and for $p = 1 - e^{-1}$. The latter is the value which makes $\epsilon_p = \alpha$ in the case of perfect reliability.

The behavior of ξ with σ is shown for several R-values in Fig. 8 for $p = 1/2$. Asymptotes are shown by short horizontal line segments. The most important characteristic of these results already has been shown by Fig. 7, namely that for R somewhat greater than p, the ξ-values just do not grow very large no matter how great σ is. In other words, even large failure severities do not degrade the performance of a relatively reliable system very much. It is noteworthy, though, that R does not need to be of the order of 0.8 or 0.9 for this to be the case, as might be expected intuitively. Even for $R = 0.55$ $(p = 1/2)$ the probable error is degraded from ϵ_{pA} only by a factor of $\xi = 1.86$

for infinite failure severity. This fact is of some consequence to the practical evaluation of $F(\epsilon)$, for it means that the form of the function is most important only in the neighborhood of $\epsilon < 2\epsilon_{pA}$, approximately.

Let us examine this latter question further by seeing how the solution changes when $A(\epsilon)$ is kept a circular normal distribution[16] but $F(\epsilon)$ is generalized to an arbitrary non-decreasing function for which $F(0) = 0$ and $F(\infty) = 1$. Assume $F(\epsilon = \alpha) \leq p$, for otherwise unreliability improves the system performance.

The analog of Eq. 8 is

$$R \exp(-\epsilon_p^2/\alpha^2) + (1 - R)[1 - F(\epsilon_p)] = 1 - p. \tag{15}$$

Replacing ϵ_p by $\alpha\xi$, Eq. 15 can be rewritten

$$1 - F(\xi) = \frac{1 - p}{1 - R}[1 - R(1 - p)^{\xi^2 - 1}]. \tag{16}$$

If the right- and left-hand sides of Eq. 16 are drawn as functions of ξ, the solution for ξ is the intersection of the two curves.

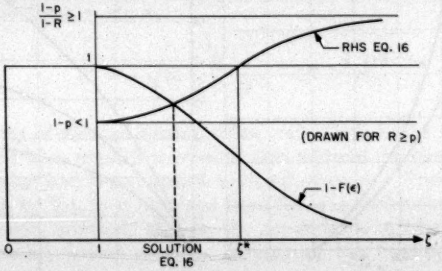


FIG. 9. Solution for general $F(\epsilon)$.

Figure 9 shows the typical behavior of the two sides of this equation. As ξ ranges over $(1, \infty)$, the right-hand side increases monotonically from $1 - p < 1$ to $(1 - p)/(1 - R) \geq 1$ (if $R \geq p$, as is assumed), crossing the value unity at the same ξ^* as defined previously by Eq. 14. Also, $1 - F(\xi)$ has a value unity at $\xi = 0$, and lies between unity and $1 - p$ at $\xi = 1$, decreasing monotonically toward zero as $\xi \to \infty$.

Under these conditions, there exists exactly one solution of Eq. 16, and this solution lies in the range $1 \leq \xi < \xi^*$. Thus, no matter what the exact form of $F(\epsilon)$, no matter how slowly it rises toward unity, the

[16] An analogous result exists, at least in qualitative terms, for generalizations of F when A itself is more general than a circular normal distribution.

system performance is bounded by the same limit as in the previously considered special case of circular normal $F(\epsilon)$ with $\sigma \to \infty$.

This result is of great importance. It means that if $R > p$, the exact form of $F(\xi)$ outside of $1 \leq \xi < \xi^*$ is of no consequence to the system performance. Furthermore, we have seen previously that for a fairly wide range of R, $\xi^* < 2$; for example, with $p = 1/2$, $R = 0.55$ implies $\xi^* \leq 1.86$. Therefore, in many cases of interest we are not concerned with $F(\epsilon)$ except over the approximate range $\alpha \leq \epsilon \leq 2\alpha$.

Moreover, inasmuch as the crossing of the curves occurs above $1 - p$, it follows that the only part of the distribution function $F(\epsilon)$ of interest in this problem is in the range $0 \leq F(\epsilon) < p$. This may correspond to a smaller ξ-range of interest than $(0, \xi^*)$, but does not necessarily. It is convenient to introduce $\bar{\epsilon}_p$ as the p-error from unreliability alone as if accuracy questions were not included.[17] That is, $F(\bar{\epsilon}_p) = p$. Then the requirement prior to Eq. 15 which prevents a performance improvement is equivalent to the requirement that $\bar{\epsilon}_p \geq \alpha$. Also, the ϵ region of interest can be represented in these terms as $\alpha \leq \epsilon \leq \min(\bar{\epsilon}_p, \alpha\xi^*)$. In most cases of interest, it seems likely that $\bar{\epsilon}_p > \alpha\xi^*$.

The fact that $F(\epsilon)$ is required only over a finite range means that it can be bounded from above and below by circular normal distributions of the type considered previously. If these bounding distributions are characterized, respectively, by failure severities σ_u and σ_l, it can be seen that ϵ_p for the general failed error distribution lies between $\epsilon_l = \epsilon_p(\sigma_l)$ and $\epsilon_u = \epsilon_p(\sigma_u)$, these ϵ-values being determined implicitly by Eq. 12. It can be hoped that this fact will be exploitable in the problem of the practical determination of $F(\epsilon)$.

11. EFFECTIVE RELIABILITY

For a given α, R and σ, Eq. 12 implicitly defines an ϵ_p. In most reliability treatments, it is considered that every failure has infinite severity. With the same R-value, we would then say that the performance of the system is a value of ϵ_p, say $\bar{\epsilon}_p$, given by

$$RA(\bar{\epsilon}_p) = p. \tag{17}$$

Thus $\bar{\epsilon}_p$ is identical with the $\epsilon_p{}^*$ of Eq. 11, and is necessarily greater than the true ϵ_p for $\sigma \neq \infty$. That is, the true performance when the non-infinite failure severity is included is better than the performance estimated with $\sigma = \infty$ and the same value of R.

This behavior is just what is meant in saying that the conventional zero-failure probability approach overstates the effect of failures—that it predicts a poorer operational value than the system really has.

But another viewpoint arises naturally in this connection. We may define an "equivalent" or "effective" reliability $\bar{R}$ as the zero-

[17] This point of view is pursued further in the next section.

failure probability value which a system would have to have when $\sigma = \infty$ in order to keep the same system performance as is actually realized with R and σ. Calling this an effective reliability can be justified on the following basis. Suppose that we have a system whose reliability is R and failure severity is σ, but that we choose to ignore the gradual degradation of performance from failures. That is, we ignore the concept of failure severity entirely. Instead, we assume that no failed systems contribute to system performance. Then we must also attribute a reliability $\bar{R}$ rather than R to the system, for it is $\bar{R}$ in combination with $\sigma = \infty$ that gives the correct system performance.

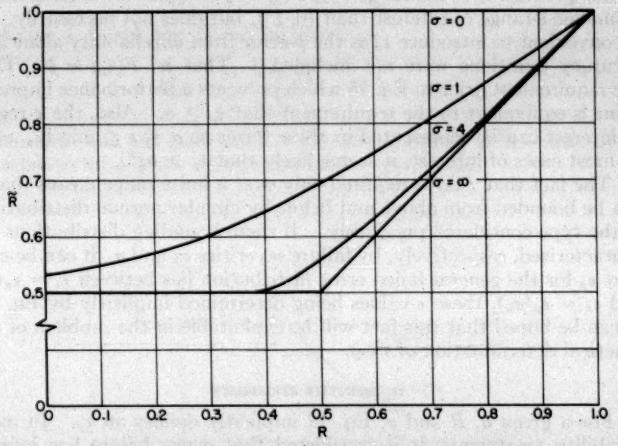


FIG. 10. Effective reliability $\bar{R}$ *versus* R for constant σ and $p = 1/2$.

It is clear that $\bar{R}$ so defined satisfies

$$\bar{R}A[\epsilon_p(\alpha, R, \sigma)] = p \tag{18}$$

with ϵ_p found from Eq. 12. Explicitly, for the special case of circular normal A and F distributions,

$$\bar{R} = p/[1 - (1 - p)^{\xi^2}] \tag{19}$$

or, eliminating ξ between Eqs. 12 and 19,

$$R = \frac{(1 - p) - (1 - p/\bar{R})^{1/(1+\sigma)^2}}{(1 - p/\bar{R}) - (1 - p/\bar{R})^{1/(1+\sigma)^2}} \tag{20}$$

in which $\bar{R} \geq p/[1 - (1 - p)^{(1+\sigma)^2}] \geq p$.

This relationship is shown in Fig. 10 for several values of σ. As would be expected, $\bar{R} > R$ for $\sigma < \infty$. The fact that the effective reliability is never less than p may at first be startling. It is a consequence of the fact that $\bar{R} = p$ is the value of $\bar{R}$ attached to a worthless system (one for which $\epsilon_p = \infty$). This behavior is inherent if the system is to have a value: as the fraction of failed systems having an infinite error approaches $1 - p$, the p-error necessarily approaches infinity. For example, if $p = 1/2$, as the system reliability drops to 1/2 (assuming all failed systems "complete misses"), the system probable error becomes infinite.

Another point of interest here is that σ need not be very large before $\bar{R}$ approaches its limiting behavior very closely, provided R is somewhat greater than p.

I believe that the principal value of the effective reliability concept is to throw light on common statements about reliability which ignore the notion of failure severity. For example, when various authors talk about the system reliability having to exceed a certain value, they usually are referring to $\bar{R}$ even when they have defined their reliability number as R is defined here—not R itself, which can be somewhat lower than $\bar{R}$ if $\sigma < \infty$ without degrading the operational value of the system.

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A NOTE ON MOHR'S STRESS CIRCLES

BY

W. D. FREESTON[1] AND E. W. SUPPIGER[1]

ABSTRACT

A simple demonstration is presented, based on the use of the generalized coordinates ϕ and θ, of the Mohr stress circle construction for the general three dimensional state of stress. Results are plotted, and tabulated, to show the relation between the two coordinates ϕ, θ of points on the element chosen and the two coordinates σ, τ of the associated points in the stress plane.

INTRODUCTION

If the stress components σ_x, σ_y, σ_z, τ_{xy}, τ_{yz}, τ_{zx} on three mutually perpendicular planes are known, one may determine the magnitudes of the three principal stresses σ_1, σ_2, σ_3 as the roots of the cubic equation (1)[2]

$$\sigma^3 - c_1\sigma^2 + c_2\sigma - c_3 = 0 \tag{1}$$

wherein the constants

$$c_1 = \sigma_x + \sigma_y + \sigma_z$$

$$c_2 = \sigma_x\sigma_y + \sigma_y\sigma_z + \sigma_z\sigma_x - \tau_{xy}^2 - \tau_{yz}^2 - \tau_{zx}^2$$

$$c_3 = \sigma_x\sigma_y\sigma_z - \sigma_x\tau_{yz}^2 - \sigma_y\tau_{zx}^2 - \sigma_z\tau_{xy}^2 + 2\tau_{xy}\tau_{yz}\tau_{zx}$$

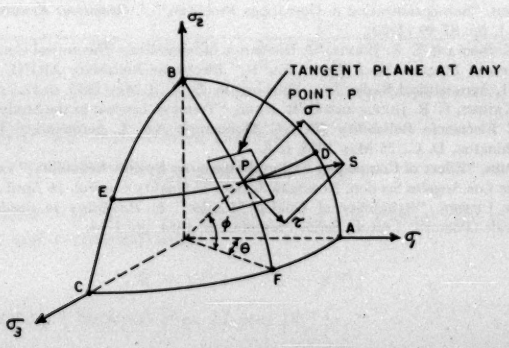

FIG. 1. Octant of spherical element, uniform state of stress, principal stresses σ_1, σ_2, σ_3.

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[2] The boldface numbers in parentheses refer to the references appended to this paper.

and the directions of the three principal stresses are likewise easily obtained (1).

Knowing the principal stresses at a point one may obtain, either analytically or graphically, the stress components σ, τ on any plane passing through the point (that is, on the tangent plane at P of the element shown in Fig. 1). A short proof of Mohr's (2) graphical solution follows using the generalized coordinates, latitude angle ϕ and longitude angle θ, of the point P.

METHOD

The coordinates of P in Fig. 1 are ϕ, θ and the coordinates of P' in Fig. 2 are σ, τ. The equations of equilibrium for the forces acting on

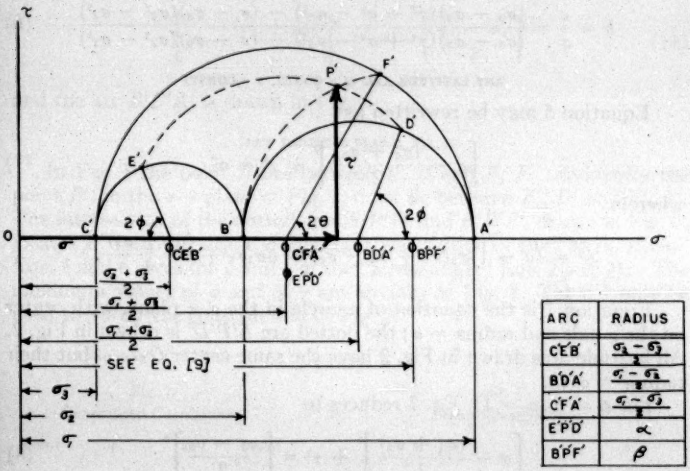

FIG. 2. Mohr's stress circles, with $\sigma_1 > \sigma_2 > \sigma_3 > 0$.

the tetrahedron formed by the three principal planes and by the tangent plane at P furnish the equations (3, 4)

$$\sigma = \sigma_1 \cos^2 \phi \cos^2 \theta + \sigma_2 \sin^2 \phi + \sigma_3 \cos^2 \phi \sin^2 \theta \tag{2}$$

$$\tau^2 = s^2 - \sigma^2 = \sigma_1^2 \cos^2 \phi \cos^2 \theta + \sigma_2^2 \sin^2 \phi + \sigma_3^2 \cos^2 \phi \sin^2 \theta - \sigma^2. \tag{3}$$

Let

$$a = \cos^2 \phi \qquad 1 - a = \sin^2 \phi$$
$$b = \cos^2 \theta \qquad 1 - b = \sin^2 \theta$$
$$c = ab = \cos^2 \phi \cos^2 \theta$$

then Eqs. 2 and 3 may be written as

$$\left.\begin{aligned}(\sigma_3 - \sigma_2)a + (\sigma_1 - \sigma_3)c &= (\sigma - \sigma_2)\\ (\sigma_3^2 - \sigma_2^2)a + (\sigma_1^2 - \sigma_3^2)c &= (\tau^2 + \sigma^2 - \sigma_2^2)\end{aligned}\right\}. \tag{4}$$

The values of a and c from Eqs. 4 are

$$a = \frac{(\sigma - \sigma_2)(\sigma_1 + \sigma_3) - (\tau^2 + \sigma^2 - \sigma_2^2)}{(\sigma_3 - \sigma_2)(\sigma_1 - \sigma_2)} \tag{5}$$

$$c = \frac{(\sigma - \sigma_2)(\sigma_2 + \sigma_3) - (\tau^2 + \sigma^2 - \sigma_2^2)}{(\sigma_1 - \sigma_3)(\sigma_2 - \sigma_1)}$$

and thus

$$b = \frac{c}{a} = \frac{(\sigma_3 - \sigma_2)(\tau^2 + \sigma^2 - \sigma_2^2) - (\sigma - \sigma_2)(\sigma_3^2 - \sigma_2^2)}{(\sigma_3 - \sigma_1)(\tau^2 + \sigma^2 - \sigma_2^2) - (\sigma - \sigma_2)(\sigma_3^2 - \sigma_1^2)}. \tag{6}$$

ANY LATITUDE ARC (ϕ = CONST, a = CONST)

Equation 5 may be rewritten as

$$\left[\sigma - \frac{(\sigma_1 + \sigma_3)}{2}\right]^2 + \tau^2 = \alpha^2 \tag{7}$$

wherein

$$\alpha^2 = (a - 1)\sigma_2(\sigma_1 + \sigma_3 - \sigma_2) - a\sigma_1\sigma_3 + \frac{(\sigma_1 + \sigma_3)^2}{4}.$$

Equation 7 is the equation of a circle in the σ–τ plane, with center on the σ-axis and radius = α; the dotted arc $E'P'D'$ is drawn in Fig. 2. All latitude arcs drawn in Fig. 2 have the same center $O_{E'P'D'}$ but their radii $\alpha = \alpha(\phi)$.

For $\phi = 0$ ($a = 1$), Eq. 7 reduces to

$$\left[\sigma - \frac{(\sigma_1 + \sigma_3)}{2}\right]^2 + \tau^2 = \left[\frac{\sigma_1 - \sigma_3}{2}\right]^2 \tag{8}$$

and the arc $C'F'A'$ is drawn in Fig. 2.

ANY LONGITUDE ARC (θ = CONST, b = CONST)

Equation 6 may be rewritten as

$$\left[\sigma - \frac{[b(\sigma_3^2 - \sigma_1^2) - (\sigma_3^2 - \sigma_2^2)]}{2[b(\sigma_3 - \sigma_1) - (\sigma_3 - \sigma_2)]}\right]^2 + \tau^2 = \beta^2 \tag{9}$$

wherein

$$\beta^2 = \frac{(b-1)\sigma_2\sigma_3(\sigma_2 - \sigma_3) + b\sigma_1\sigma_2(\sigma_1 - \sigma_2)}{(b-1)\sigma_3 - b\sigma_1 + \sigma_2} + \frac{1}{4}\left[\frac{b(\sigma_3^2 - \sigma_1^2) - (\sigma_3^2 - \sigma_2^2)}{b(\sigma_3 - \sigma_1) - (\sigma_3 - \sigma_2)}\right]^2.$$

Equation 9 is the equation of a circle in the σ–τ plane, with center on the σ-axis and radius $= \beta$; the dotted arc $B'P'F'$ is drawn in Fig. 2. For any longitude arc drawn in Fig. 2, both the abscissa of the center and the radius β are functions of b (that is, of the longitude θ).

For $\theta = \frac{\pi}{2}$ ($b = 0$), Eq. 9 reduces to

$$\left[\sigma - \frac{(\sigma_2 + \sigma_3)}{2}\right]^2 + \tau^2 = \left[\frac{\sigma_2 - \sigma_3}{2}\right]^2 \tag{10}$$

and the arc $C'E'B'$ is drawn in Fig. 2.

For $\theta = 0$ ($b = 1$), Eq. 9 reduces to

$$\left[\sigma - \frac{(\sigma_1 + \sigma_2)}{2}\right]^2 + \tau^2 = \left[\frac{\sigma_1 - \sigma_2}{2}\right]^2 \tag{11}$$

and the arc $B'D'A'$ is drawn in Fig. 2.

ANY TANGENT PLANE, $P(\phi, \theta)$

In Fig. 1 the point P lies between E, D and B, F. Evidently the point P', in the σ–τ plane of Fig. 2, must lie between E', D' and B', F'. The intersection of the dotted arcs $E'P'D'$ and $B'P'F'$, drawn in Fig. 2, locates P' (analytically, the simultaneous solution of the "angle-stress" Eqs. 5 and 6 gives the σ and τ of the "stress-angle" Eqs. 2 and 3). The maximum values of σ and of τ are obvious in Fig. 2. Table I may be used to verify the construction of Fig. 2.

TABLE I.

Fig. 1. Generalized Coordinates			Fig. 2. Stress Coordinates		
Point	ϕ	θ	Point	σ	τ
A	0	0	A'	σ_1	0
B	$\frac{\pi}{2}$	0	B'	σ_2	0
C	0	$\frac{\pi}{2}$	C'	σ_3	0
D	ϕ	0	D'	$\frac{(\sigma_1+\sigma_2)}{2}+\frac{(\sigma_1-\sigma_2)}{2}\cos 2\phi$	$\frac{(\sigma_1-\sigma_2)}{2}\sin 2\phi$
E	ϕ	$\frac{\pi}{2}$	E'	$\frac{(\sigma_2+\sigma_3)}{2}-\frac{(\sigma_2-\sigma_3)}{2}\cos 2\phi$	$\frac{(\sigma_2-\sigma_3)}{2}\sin 2\phi$
F	0	θ	F'	$\frac{(\sigma_1+\sigma_3)}{2}+\frac{(\sigma_1-\sigma_3)}{2}\cos 2\theta$	$\frac{(\sigma_1-\sigma_3)}{2}\sin 2\theta$
P	ϕ	θ	P'	Eq. 2	Eq. 3

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TOMORROW'S AGRICULTURE—THE RESEARCH PERSPECTIVE*

BY

BYRON T. SHAW[1]

It is a great privilege to be here tonight. I am deeply honored to be chosen as the recipient of the 1959 Agricultural Award of the Philadelphia Society for Promoting Agriculture. I am confident, however, that your distinguished Society, in making its choice, was thinking of all the research scientists in the Department of Agriculture and their many contributions to the nation's welfare. It therefore seems to me appropriate to accept the Award on behalf of my colleagues, as well as myself. We are gratified to receive this recognition from a group so eminent in its support of agricultural science.

The medal symbolizing The Philadelphia Agricultural Award, presented to Dr. Byron T. Shaw by the Philadelphia Society for Promoting Agriculture, on April 22, 1959.

I appreciate also the opportunity to participate in this joint meeting of the Society and The Franklin Institute. It is always a pleasure for me to discuss agricultural research with people who have a real and continuing interest in scientific progress.

When Colonel Biddle invited me to speak tonight, he didn't specify what I should talk about. Considering our mutual interests, I should like to examine with you a perspective of our future agriculture as it looks to an agricultural scientist. And for this purpose, I should like to

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proceed mainly from the viewpoint of basic research, and to consider its potential impact on agriculture during the next half century.

I recognize that you are keenly aware of the history of our agriculture, and are familiar with the steps by which this country has advanced to a position of world leadership in farm production. But since history helps us get a proper perspective, I should like to examine briefly the major forces responsible for our agricultural progress so far.

The history of farming in this country seems to be divided broadly into three periods. The first—extending from colonial times through World War I—was a period of physical growth by the development of new lands. In general, total farm output during this first 300 years increased only as additional cropland was put under the plow. Acre for acre, crop *yields* remained about the same.

Fortunately for our country, there were farsighted individuals during this early period who knew that good farmland would one day all be farmed, and that if we were to build a great nation, something had to be done to increase farming efficiency. The Philadelphia Society for Promoting Agriculture was the first organized group to tackle this problem. Its efforts and those of other like-minded groups brought about the establishment, 100 years ago, of the Land Grant Colleges and the U. S. Department of Agriculture.

V.2
195

Research was encouraged, and new farm practices began to emerge. But even fifty years later, crop yields on the average remained the same. Many changes in land use occurred that should have improved yields. Vast areas of highly fertile virgin land were plowed up, and worn-out areas were discarded. Millions of acres of potentially productive wet land were drained. Fertilizer and lime use increased to substantial quantities. New, higher yielding crop varieties were introduced and controls were developed for a number of insect pests and crop diseases. Yet with all these improvements, yield levels stayed about the same.

There was only one possible conclusion. All the improvements in farming that had been made had barely succeeded in offsetting the decline in soil productivity that was taking place.

Public concern over this situation was mounting by 1908, when President Theodore Roosevelt convened the first Governors' Conference to consider resource problems. Out of this conference emerged the conservation idea.

The second period in our farming history—covering roughly the years between the two World Wars—is notable for two developments. The first of these was the application of mechanical power in farming, which gradually released millions of acres from the production of feed for horses and mules. These acres became available for food production. The second important development was the action taken, on a broad front, by the Federal Government, by the States, by industry, and by farmers, to improve our agriculture. Research was given due recognition.

The third period in our farming history is the one we are in now. Today we can see the fruits of the efforts started in the earlier periods.

In 1939, when World War II broke out in Europe, American farmers produced a 2½ billion-bushel crop of corn on 88 million acres. Last year, they produced a record 47 per cent more on 15 million fewer acres. The story repeats itself with virtually all major crops. The 740 million bushels of wheat produced in 1939 took 52½ million acres. Last year, on about the same acreage, the crop was 1 billion 460 million bushels, virtually double that in 1939. Production of oilseed crops has almost tripled since 1939.

It is the same with livestock. In 1958 we had nearly 3½ million fewer dairy cows than in 1939, but each cow produced seven-eighths of a ton more milk during the year. For every two eggs a hen laid in 1939, her descendant is laying about 3 eggs today. Total egg and poultry production is up 108 per cent. We have 60 million beef cattle and calves on the same pastures and range lands that in 1939 supported only 30 million head. We had a pig crop of 95 million in 1958 on the same farm plant that produced 87 million in 1939.

All told, we produced 54 per cent more farm commodities last year on fewer acres than we had in 1939.

But these facts and figures alone don't tell the whole story. Figures on manpower required to do the job also are significant. In World War I, we produced our farm commodities with 13½ million workers; in World War II, with 10½ million workers; today there are only 7½ million farm workers.

If the agricultural output we achieved in 1957—the latest year for which we have complete figures—had been produced by the methods available to farmers in 1939, it would have cost the nation about 7½ billion dollars more in land, labor, capital, and other resources than the actual cost in 1957. These 7½ billion dollars were therefore available for other improvements in our living standards.

The story is not all bright, however. From the economic standpoint, as you well know, farmers have not shared equally with the rest of us in the progress our country has made during the past two decades. They benefited greatly from the adoption of technological improvements when the markets were expanding during World War II and the rehabilitation years. Their purchasing power rose rapidly, and they began to pay debts, to buy land, livestock, and equipment, and to make many farm and home improvements. But as the general level of prices rose, it brought rising costs for farm labor, machinery, and production supplies of all kinds. Then, when the special needs of war and rehabilitation had been met, prices received by farmers began falling, and surpluses began piling up. Thus, in recent years, farmers have been caught in a cost-price squeeze. And the burden of farm surpluses has been felt by the entire economy.

Today our most urgent problem is to find new ways of expanding markets for the abundance now produced on our farms.

Research must develop new industrial uses for agricultural commodities. At the same time, it must give farmers new techniques for producing commodities of uniform quality in high volume—for producing them in ways that are profitable to farmers and at costs that permit competition with other industrial raw materials.

And now let us look to the future.

We are all aware of the present rapid increase in our population and the predictions that this trend will continue. The Census Bureau estimates that by the year 2010 we may have 370 million people—more than twice the population we have today.

This means that just to maintain our present diet levels, we will require twice as much food and other farm products as we are consuming today. New knowledge of nutritional requirements, especially for older and younger age groups, is emphasizing the need for more protective foods—those high in protein, vitamins, and materials. Meat, milk, eggs, and fruits and vegetables provide these requirements, but they are also the foods with high production and processing costs. To make our people fifty years from now as well fed as they should be, farmers will have to at least double their present crop output and more than double present production of livestock products.

At the same time, the amount of farm land available is not likely to be increased much beyond the acreage farmers are using today. Some new land can be brought into production by various methods. But, as our population increases, considerable present farm land will go into urban and other non-farm uses. Trends also indicate that our farms will continue to increase in size and decrease in numbers, and that additional farm workers will seek part- or full-time employment in towns and cities.

In summary, then, we can expect that tomorrow's farmers—with only a little more land and considerably less manpower—will have to produce for a rapidly increasing population, whose needs and desires will influence, more and more, the kinds and quantities of products produced. Despite our present abundance, these demands will not be met unless ways are found to increase efficiency further throughout agriculture.

Farmers will have to do a better job of conserving soils and using available water supplies. They will need higher yielding strains of crops and livestock with specific qualities to meet special market demands—lean, tender beef, for example—milk with more solids and less fat . . . eggs that retain their initial high quality . . . fruits and vegetables more suitable for freezing and canning . . . field crops with qualities especially useful to industry. Farmers must have more economical and effective methods of controlling diseases, insects, weeds, and weather . . . better fertilizer practices, machines, and other production tools. And they must be able to fit these improvements to-

gether into economical farm operations that are flexible enough to allow adjustments in response to changes in market demands.

Furthermore, agricultural efficiency no longer stops at the farm gate. It extends into the processing plant, the retail store, and the home—wherever farm products are ultimately used. It means maintaining the quality of products after they leave the farm. It means efficient and economical methods of handling, processing, and distribution. And it means efficient utilization of all agricultural commodities, whether as industrial raw materials or as consumer end-products.

All these things contribute to total agricultural efficiency. And the only way that I know they can be achieved is through agricultural research—pursued vigorously and steadily by both public and private agencies.

In the Department of Agriculture, we have been concerned with this problem for some time. We have been giving a great deal of thought to the kind of research that will help us to make the most progress over the long term.

We have become convinced that our greatest need is for basic research to discover new principles and new methods that will help us to understand fundamental biological processes.

We are fortunate to have seen in our own lifetime how basic research in the physical sciences has given man new power to manage molecules and new insight even into the nucleus of the atom. We are now in the golden era of the physical sciences. The next golden era in science will be in the biological sciences. It will come as we gain understanding of the cell as the unit of life.

The findings of this research promise to rival in importance anything that man has ever done. They will be particularly important to agriculture. If we can better understand and control the mechanisms and functions of living cells, we will have vastly increased ability to breed more productive, higher quality crops and livestock . . . to control or eradicate diseases and insect pests . . . to maintain the quality of farm products during processing and marketing . . . to find new uses for farm-grown raw materials . . . and to improve human nutrition.

Let us examine some of the puzzles we face in our efforts to understand the workings of cells.

Anyone who has grown a plant knows that nutrient elements, such as potassium, pass readily from the soil into the plant. It may surprise some of you to learn that no one knows how this takes place.

The mystery is that, in the soil, nutrient elements occur typically in solutions having low concentrations. Inside the plant, however, the concentration of the same nutrients is typically high. And yet plant nutrients are regularly transferred from the area of low concentration in the soil to the area of high concentration in the plant.

If we can find out why this "up-hill" flow occurs, it will give us new

insight into plant growth and undoubtedly will lead us to more efficient methods of fertilizing crops.

Many of you have had the experience of seeing land that has not been plowed for some years suddenly show a full cover of weeds when it was plowed, even though none of the weeds had grown on this land for years. You have probably asked how weed seeds could stay dormant in the soil for so long—sometimes for as much as twenty years—and then, following the plowing, suddenly germinate.

We now know that exposure to light in the plowing process, even though only for a few seconds, is responsible for triggering the germination. Likewise, light controls flowering, stem elongation, pigment formation, tuberization, and many other growth processes.

Here again we have the unanswered question of how light regulates these phenomena. As a first step we need to know the chemistry of the active material in the cell that absorbs the light. Knowing that, we can take another step.

When we fully understand what light does in seed germination, we may find a way to make all weed seeds in the soil germinate at once, in the dark, and then quickly get rid of the weeds.

If we could unravel some of the chemical processes occurring within living cells, we would be much nearer to an understanding of the fundamental nature of life.

How do viruses reproduce themselves, when other large protein molecules not containing nucleic acid cannot do so? What is the nature of the chemical or physical changes which result in virus mutations? If we had the answers to these questions, we might be able to produce mild strains of many viruses capable of immunizing plants and animals against diseases caused by more virulent strains. Or we might develop chemical methods for destroying the viruses.

A part of this riddle is that all proteins are composed of the same amino acids combined in various ways and proportions. Yet one protein turns out to be inert, another with nucleic acid is a virus, and still another is an enzyme. And enzymes are the controlling catalysts in metabolic processes.

There are many different enzymes—at least one for each process. Yet there is an underlying unity in the molecules that nature uses for its processes. Similar enzyme molecules catalyze similar reactions both in one-celled organisms and in higher animals. This gives us hope that in our studies of plants and animals and micro-organisms, we will one day be able to fit the pieces of our jig-saw puzzle together into an understanding of the nature and behavior of enzymes and viruses and, in fact, of all proteins.

This knowledge will help us to understand metabolic processes in cells. It could well turn out that major errors in cell metabolism may be involved in cancer, and that the accumulative effect of minor metabolic errors or imbalances may contribute to the aging process. If we

can learn more about cell metabolism and the cause or control of errors in this basic life process, it could benefit not only agriculture but all mankind.

A question that has long troubled biologists is what makes one cell reproduce itself and eventually become a muscle, while other cells develop through a similar process into nerves, fat, cartilage, or bone? It seems likely that genes and cytoplasmic particles play a major role in both development and differentiation. This leads to the question of what is the nature and structure of the fundamental unit of heredity, and how does it produce its effect?

Recent evidence indicates that the gene may not be the fundamental unit of heredity, in terms of composition, but may consist of chemical sub-units. An understanding of the chemical organization of the fundamental units, how they reproduce themselves, and how they produce their effects in the organism would provide valuable new methods of genetic control in plant and animal populations.

If we can understand the chemistry of genes, we may learn how to modify them by chemical treatment. From this it follows that metabolic processes would become subject to modification.

The implications of such mastery over biological behavior are tremendous. If specific desirable changes could be made, the rate and amount of genetic improvement in animal and plant populations would far exceed anything now possible.

I have only touched on a few of the questions research must answer if our agriculture is to meet the demands that will be placed upon it fifty years from now and in the longer future. There are many others—dealing with every phase of agriculture. And involved in all of them are the problems farmers will face in adjusting to the changes that will come because of technological developments—both on and off the farm. This means that we must do much more research also in the economics of production, utilization, and marketing—so that we can give farmers the information they need to make the right decisions in adjusting their operations in light of market demands.

The success of research in helping agriculture meet the needs of the year 2010—or of the time between now and then—depends on many things. But certainly the most important is the support given to basic research. I am exceedingly pleased that both in the Land Grant Colleges and in the Department of Agriculture great strides have been made in recent years to strengthen our basic work. It now accounts for more than 20 per cent of our total effort. But more still needs to be done.

I should like to suggest that the Philadelphia Society for Promoting Agriculture, which has such a long and distinguished record of supporting agricultural science in the past, might become the champion of basic research, the key to a greater agriculture for the future.

WAVE MECHANICS FOR HYDROGEN ATOM IN CYLINDRICAL COORDINATES: NON-SEPARABLE EIGENSOLUTIONS

BY

LOUIS GOLD[1]

ABSTRACT

The solution of the hydrogen atom in cylindrical coordinates serves to demonstrate the manner in which non-separable differential equations can be treated. The eigenvalues for the H atom are so deduced in a simpler fashion and the correspondence to the conventional spherical coordinate results is pointed out. In this manner the important matter of degeneracy is entered into; the quest for non-separable solutions must be characterized by appropriate attention to this detail.

INTRODUCTION

In the solution of eigenvalue problems, it is customary (as for the hydrogen atom) to transform to a coordinate system in which the spatial variables are completely separable. It is of course well known that the Schrödinger equation for the H atom is separable in parabolic (1)[2] as well as spherical coordinates; indeed, the former system may be employed to advantage in certain problems (2).

The present report offers testimony to the fact that it may not be essential to separate variables to secure satisfactory solutions. In dealing specifically with the H atom in cylindrical coordinates, it was discovered that the energy levels can so be found expeditiously. It turns out, however, that a straightforward connection between the cylindrical and spherical coordinate solution exists.

As an interesting sidelight, it was later found that Weston (3, 4), only recently, was concerned with the problem of finding a class of non-separable solutions and actually provided a solution for toroidal wave functions. Thus it would appear that many heretofore so-called insoluble eigenwert problems deserve a re-examination with the view of extracting non-separable solutions. The following analysis will indicate how such an approach proceeds.

FORMULATION OF THE H ATOM IN CYLINDRICAL COORDINATES

The wave equation for the hydrogen atom is the familiar

$$\frac{\hbar}{2m_0}\nabla^2\psi + \left(-W + \frac{Ze^2}{r}\right)\psi = 0, \quad W > 0 \tag{1}$$

for which only negative energy states are sought that correspond to

[1] Edgerton, Germeshausen and Grier, Inc., Boston, Mass.

[2] The boldface numbers in parentheses refer to the references appended to this paper.

bound states; positive energy states relate to imaginary coordinates and are ignored. It is convenient to express (1) in dimensionless form by putting

$$\left.\begin{aligned} x &= \alpha u \\ y &= \alpha v \\ Z &= \alpha w \end{aligned}\right\}, \quad \alpha^2 = \hbar^2/2m_0W \tag{2}$$

whereby

$$\left(\frac{\partial^2}{\partial u^2}+\frac{\partial^2}{\partial v^2}+\frac{\partial^2}{\partial w^2}\right)\psi+\left(-1+\frac{\epsilon}{\sqrt{u^2+v^2+w^2}}\right)\psi=0 \tag{3}$$

$$\epsilon=\frac{Ze^2}{\hbar}\left(\frac{2m_0}{W}\right)^{1/2}.$$

Now the introduction of cylindrical coordinates via

$$u = u, \quad w = \rho\cos\phi, \quad v = \rho\sin\phi \tag{4}$$

leads to

$$\left(\nabla^2-1+\frac{\epsilon}{\sqrt{u^2+\rho^2}}\right)\psi=0 \tag{5}$$

where

$$\nabla^2=\frac{1}{\rho}\frac{\partial}{\partial\rho}\left(\rho\frac{\partial}{\partial\rho}\right)+\frac{1}{\rho^2}\frac{\partial^2}{\partial\phi^2}+\frac{\partial^2}{\partial u^2}. \tag{5a}$$

Elimination of the variable ϕ is effected by

$$\psi = e^{im\phi}\chi \tag{6}$$

when introduced into (5). Thus

$$\Phi=\frac{1}{\sqrt{2\pi}}e^{im\phi}, \quad m = 0, \pm 1, \pm 2, \cdots$$

with the remaining two variable equations for $\chi(u, \rho)$:

$$\left[\frac{\partial^2}{\partial\rho^2}+\frac{1}{\rho}\frac{\partial}{\partial\rho}+\frac{\partial^2}{\partial u^2}-\left(1+\frac{m^2}{\rho^2}\right)+\frac{\epsilon}{\sqrt{u^2+\rho^2}}\right]\chi=0. \tag{7}$$

This cannot be further separated.

SOLUTION FOR $\chi(u, \rho)$ IN TWO VARIABLES

The conventional view about non-separability would preclude any further effort to solve the H atom in this manner. Careful study of (7) reveals, however, that solutions of the form

$$\chi = \text{const}\cdot\rho^n e^{\beta(\rho^2+u^2)^{1/2}} \tag{8}$$

are appropriate, provided

$$\beta^2 = 1, \quad n = m, \quad 2\beta(n+1) + \epsilon = 0 \tag{9}$$

as direct substitution will testify. Thus the eigenvalues are secured at once since from (9)

$$\epsilon^2 = 4(n+1)^2$$

so that explicitly

$$W = \frac{Z^2 e^4 m_0}{2\hbar^2(n+1)^2}, \quad n = 0, 1, 2, 3, \cdots. \tag{10}$$

Clearly, bounded values of χ require $\beta = -1$ which provides the final wave function relation

$$\psi = \text{const} \cdot \rho^n e^{-(\rho^2+u^2)^{1/2}} e^{in\phi}. \tag{11}$$

Normalization defines the value of the constant in the usual way

$$\int_{\rho=0}^{\infty}\int_{u=0}^{\infty}\int_{\phi=0}^{2\pi} \psi\psi^* \rho d\rho du d\phi = 1. \tag{12}$$

THE CONNECTION WITH SPHERICAL COORDINATE SOLUTION

In proceeding with the normalization, there is encountered

$$c^2 \int_{\rho=0}^{\infty}\int_{u=0}^{\infty} \rho^{2n+1} e^{-2(\rho^2+u^2)^{1/2}} d\rho du = \frac{1}{2\pi} \tag{13}$$

where c is the normalization constant. The evaluation of (13) is expedited by introducing polar coordinates

$$\left.\begin{aligned} \rho &= r\sin\theta \\ u &= r\cos\theta \end{aligned}\right\}, \quad r^2 = \rho^2 + u^2. \tag{14}$$

But this reverts back precisely to the spherical coordinate solution

$$\psi(r, \theta, \phi) = cr^n \sin^n \theta e^{-r} e^{in\phi} \tag{15}$$

whence it emerges that the solution (11) reduces to

$$\psi = cr^n e^{-r} P_i^n, \quad n = l \tag{16}$$

with P_i^n the correlated spherical harmonics.

Clearly then, the solution of form (8) corresponds to only one of the S states; states with different radial solutions for the same angular solution are not contained in (8). The complete set of solutions in

cylindrical coordinates should presumably result when appropriate linear combinations of (11) are taken.

This degeneracy question is not unexpected since it will occur whenever the wave equation can be solved in a number of ways. If this were not so, the wave functions obtained in the various coordinates would need to be identical except for a multiplying constant—a situation not very likely to occur. Only the $n = 0$, or ground state, avoids this complication.

Hence in searching for non-separable solutions of the wave equation (and related equations) particular care must be exercised to allow for degeneracies.

DISCUSSION AND CONCLUSION

While the foregoing analysis does not contribute any new information about the hydrogen atom and is essentially quite elementary, it does demonstrate the applicability of non-separable solutions and should serve to promote a broader approach to wave mechanical and related problems. Together with the work of Weston, the present report should tend to de-emphasize the quest for separability as a precursor to securing answers to eigenvalue problems. A wholesome regard for degeneracy complications must be maintained to assure the attainment of the complete set of solutions.

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V.2
195

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BOOK REVIEWS

HEAT TRANSFER ENGINEERING, by Wilbert Schenck, Jr. 310 pages, diagrams, 6 × 9 in. New York, Prentice-Hall, Inc., 1959. Price, $9.25.

This elementary text achieves a freshness of presentation of the subject's engineering aspects which reflects the author's practical experience in the field.

In a departure from conventional arrangement of subjects dealt with in similar works, such topics as source-and-sink solutions, field plots, numerical solutions and analogies are all discussed in the second chapter on steady-state conduction, thus emphasizing the common aspects in the diversity of methods used to solve this type of problem. Free and forced convection are fully treated in the third and fourth chapters, beginning with discussion of dimensional analysis, and a detailed analysis of convective heat transfer from a flat plate and ending with an outline of thermal problems in high speed gas flow.

The fifth chapter, on radiative heat transfer, is particularly successful in clearly presenting the geometry of angle factors. (However, in the derivation of the Earth's temperature, the Earth's diameter is erroneously shown as 4000 miles on p. 124.) Chapter 6 deals with unsteady state problems involving both conduction and convection, with discussion of solution by analogies, both electrical and hydraulic, and by the Schmidt Finite Difference Method. The final chapter in the first, or methodological part of the book is concerned with combined heat transfer problems, ending with an analysis of fins, and thus providing a transition to chapter 8 on boilers and condensers, and chapter 9 on heat exchangers.

The tenth chapter is descriptive in nature and shows the applicability of the methods previously discussed to a number of systems of current interest, such as turbo jet and rocket engines, nose cones, nuclear reactors, solar heating plants and heat pumps. A chapter on methods and equipment for heat transfer testing concludes the book, and a number of tables and monograms are included in an Appendix.

The diagrams are uniformly well drawn, without the elaborateness which has become a feature of many recent textbooks, to the detriment of clarity. References, many of them to recent publications, are given at the end of each chapter. The author is to be especially commended for his selection of the many problems, which include a generous proportion of the type encountered in actual engineering practice, where a clear preliminary analysis of conditions is at least as important as the insertion of correct numerical values into handbook formulas. In keeping with this, some of the problems given in the final chapter are intended to suggest actual research projects. Throughout, emphasis is placed on obtaining solutions in the simplest possible forms, with more elaborate mathematical methods left for advanced work.

As an introduction to the engineering aspects of heat transfer, this book may be highly recommended.

K. L. CAPPEL
The Franklin Institute Laboratories

COMPUTABILITY & UNSOLVABILITY, by Martin Davis. 210 pages, 6 × 9 in. New York, McGraw-Hill Book Co., Inc., 1958. Price, $7.50.

This book, part of its publisher's series in "Information Processing and Computers," deals with a mathematical theory that could justifiably be called the foundation of all computation and decision-making methods. This theory includes several aspects: recursive functions, algorithms, Turing machines, decision problems, etc.

A mathematical function is said to be computable (or effective, or recursive) if there exists an algorithm to calculate the value of the function given its arguments; an algorithm is a set of rules which prescribes a computation in such a way that, given the result of any step of the computation, the procedure to be followed for the next step is fully and mechanically specified by the rules. A mathematical

theory is said to be decidable (or the decision question of the theory is said to be solvable, or there is a decision procedure for the theory) if there is an algorithm which can answer every yes-no question of the theory.

The book begins with a mathematically precise definition of the concept of computability in terms of Turing machines. These are like digital computers except that they have an ideal uniform structure for mathematical study; being ideal entities they have no physical existence but exist only in lists of rules of operation. Those who work with computers may regret that after two chapters on Turing machines the author relates this concept of computability to other mathematically precise concepts and that thereafter there is little trace of the Turing-machine concept. In the reviewer's opinion, however, this is as it should be (since these other concepts are just as important for computer application), except that the uninitiated reader could profit by more examples of Turing machines. The actual specification of a universal Turing machine would be quite useful, because of its similarity to a general purpose computer, being capable of being "programmed" to compute any computable function. The notions of *recursive* and *primitive recursive* are introduced in the third chapter, the arithmetization (via Godel numbers) of Turing-machine theory is given in the fourth, and decision problems are discussed in the fifth. This much (about 80 pages) concludes part I on the general theory. The remainder of the book (about 110 pages) concerns applications and further developments of the general theory. It contains chapters on production systems, diophantine equations, mathematical logic, the Kleene hierarchy, computable functionals and the classification of unsolvable decision problems. There is a short appendix containing some results from the elementary theory of numbers, and several pages of references.

The book is well written and reflects the competence of the author who has contributed significant results to the field. However, the reviewer finds, from his own experience teaching from the book, that it is not suitable as a text for the uninitiated. Through most of the book there is simply not enough motivation for the student who has no previous exposure to the field. The student who also lacks any work in formal logic is even more at a loss. There is a great tendency to have too many defined concepts, staggering the memory and concentration of the student who is not able to distinguish between those concepts which are important for the theory as a whole, and those which are merely temporary expedients in exposition.

The reviewer thinks that it would have been useful to include the old definition of general recursive as given by Kleene (namely, the one given in terms of sets of equations) instead of the one given on page 41. The importance of the old definition can be made plausible to the uninitiated reader as an explication of the idea of an effective function, whereas the definition on page 41 in terms of composition and minimization can not.

On the other hand, the reviewer feels most enthusiastic about this book as a reference for research workers in the field and as a text for advanced courses. It is a fairly complete treatment of the field, with a precise and lucid development.

ROBERT MCNAUGHTON
University of Pennsylvania

CATALYSIS, VOL. VI, edited by Paul H. Emmett. 706 pages, diagrams, 6 × 9 in. New York, Reinhold Publishing Corp., 1958. Price, $19.50.

This volume of the series has been designated as "hydrocarbon catalysis" and covers the theories and practices conducted primarily in the petroleum industries.

"Catalytic Alkylation of Paraffins with Olefins" in Chapter I has been treated by Dr. R. M. Kennedy in a concise manner with many comprehensive tables for a relatively short chapter. His presentation of the prevailing mechanisms is given in a clear fashion.

Professor F. E. Condon has condensed the very broad subject of "Catalytic Isomerization of Hydrocarbons" (Chapter 2) into a consideration of the theoretical and physical chemical aspects of the catalytic isomerization of five classes of hydrocarbons. These are: alkanes, alkenes, alkynes, naphthenes, and aromatics. The discussions of the various mechanisms for the different types of catalysts in use are amply referenced for more detailed study by the reader. The extent of the

author's efforts may be noted by the scope of over 300 references.

"Polymer Formation and Decomposition," by Professor R. Simha and Dr. Leo A. Wall (Chapter 3) is a theoretical treatment of the mechanisms involved in polymerization processes. Free radical, ionic, photo and thermal initiation are discussed with many tables of supporting data. Logically, the mechanisms of degradation are included. These authors have also been very generous in the notation of 374 references.

"Polymerization of Olefins" to liquid monomers (Chapter 4) by Drs. A. G. Oblad, G. A. Mills, and Heinz Heinemann is a major factor in present day petroleum processing. While basic mechanisms discussed in this chapter overlap with other chapters, the treatment is specific to the classes of liquid and gaseous hydrocarbons and essential to a condensed volume of this nature.

One of the first major processing improvements in the petroleum industry was that of "Catalytic Cracking" (Chapter 5). Dr. Hervey H. Voge has surveyed this subject from both the theoretical as well as practical aspects. While the catalytic cracking reactions are complex, the carbonium ion theory is shown to explain them in a qualitative sense. Additional research is required to develop a quantitative picture of the mechanism.

The final chapter on "Catalytic Reforming of Pure Hydrocarbons and Petroleum Naphthas," by Dr. F. G. Ciapetta, R. M. Dobres, and R. W. Baker encompasses the most important economic development in the petroleum industry in the past fifteen years. The authors have included much up-to-date information, even some unpublished work of timely interest. While the art is far advanced of the science in this area, the extensive operating data will be very valuable to researchers in the field.

Professor Emmett and his collaborating authors have assembled another volume of considerable scientific value to the advanced student as well as the research worker. The authors have brought together a wealth of industrial experience for the reader's benefit in a uniformly well presented fashion.

R. S. DALTER
Spencer Chemical Company

TARGET FOR TOMORROW, by I. M. Levitt. 328 pages, plates, diagrams, 5½ × 8 in. New York, Fleet Publishing Corp., 1959. Price, $4.95.

Dr. Levitt, who is an unusual combination of astronomer, teacher, and missile scientist, has covered the entire spectrum of space research, planned and coming, in his book. His first three chapters cover with great accuracy the basic astronautical facts the lay reader should know before going on to the other more exciting areas of satellites and space stations. His Chapter 3 suffers a little from an inadequate treatment of the reaction principles and the principles of the step rocket. I believe here Dr. Levitt was trying to cram too much into one chapter, or rather too little into a short chapter.

Again, in his "Rocket Power for the Future," Chapter 4, one wishes he could have treated the ion rocket in more detail, but perhaps I am looking at this from the standpoint of a worker in the field, and not a lay reader. There is no question that ion propulsion will form a significant part of future deep space propulsion efforts. His discussion in later chapters of the weather satellite is most pertinent since NASA and ARPA are interested in such payloads and have indeed let several contracts.

Similarly, his chapter on "The Observatory in Space" is very useful, showing how the atmosphere impedes the astronomer for good vision; therefore, this discussion is certainly in order, particularly since several space probes are but in effect miniature observatories. In Chapter 10, his description of the space suit is the first I have seen in a book of this kind and gives the reader intimate detail of the problems a space traveler must face.

In the last six chapters he has boldly plunged on into the future showing how man can land on the moon and establish a closed cycle there. I am sorry Dr. Levitt did not treat motions by vehicles on the moon in some detail and also problems of radiation protection and communication with the Earth. One of the great problems of moon inhabitants will be travel about the surface of the moon either in jet platforms or non-air-breathing tractors.

Dr. Levitt, who has for years discussed the problems of interstellar travel, shows clearly

that the theory of relativity may provide an answer to travel to the nearest stars because of "time dilation." However, I believe in the latter discussion Dr. Levitt is in error when he states that the human processes will slow down; actually, in a space ship traveling near the speed of light, no physical effects will be noted by the travelers. I believe Dr. Levitt was trying to say this, but a little unclearly. His conclusion at the end that a new educational approach to the training of "space engineers" is a sound one and his treatment of the curriculae needed for the space age is very timely. He has been too modest to add that for many years he has been a chief exponent of space education; however, at long last universities and secondary educational institutions are realizing the significance of the space age.

K. R. STEHLING
NASA

MATERIALS FOR ROCKETS AND MISSILES, by Robert G. Frank and William F. Zimmerman. 214 pages, illustrations, 5 × 7¼ in. New York, The Macmillan Co., 1959. Price, $4.50.

In the new industry of rockets and missiles much demand has been created for materials to meet the need of this new technology. This new science has created an unprecedented demand for materials with high melting points, high thermal conductivity, incredibly good insulators and completely shock proof. While these materials have been with us for a long time, their infrequent application and thus knowledge of their characteristics have been limited. Therefore to most engineers they were virtually unknowns.

To meet the needs for a compendium of these exotic materials a new book *Materials for Rockets and Missiles* by two engineers at the Flight Propulsion Laboratory of the General Electric Company in Cincinnati has been written. It admirably serves this purpose.

The authors have taken materials such as the sheet alloys with an iron, nickel or cobalt base used in the fabrication of rocket motors and have derived a complete description of their properties along with graphs showing their elongation, stress-rupture strength, tensile strength, etc. They have also done this for the wrought and cast alloys for the same bases. They cover the exotic alloys involving molybdenum, titanium, magnesium, etc. Even the use of ceramics is covered.

An excellent bibliography completes the book.

With the tremendous flux in this field it is certain that in a short time the material contained therein will be dated. However, until that time this book will serve a most useful purpose in providing a source book for the engineer and designer.

I. M. LEVITT
The Franklin Institute

V.2
195

BOOK NOTES

MAN . . . IN THE UNIVERSE, by Michael Stoiko and Donald Cox. 42 pages, illustrations, 7¼ × 9¼ in. Philadelphia and Toronto, The John C. Winston Co., 1959. Price, $2.95.

There are many who would like to learn a good bit more concerning the solar system and the universe beyond. In most cases when we look for something like this, it is found wrapped up in such a forbidding array of mathematical symbolism and abstruse figures that discouragement is instantaneous. There should be other books—primers—which forego the formalism of the textbook and try only to be descriptive. Only occasionally will such a book appear. *Man . . . in the Universe* is an excellent example of such a primer.

This is one of the most attractive little books dealing with the solar system with a space travel flavor. The authors are prominent men in their field who have combined skills to produce a most readable and informative book. Each page of text is accompanied by a brilliant picture superbly rendered by an accepted master of the space age art, Nick Stanilla. The combination of text and art is precisely the right mixture for the busy life we lead today. For a true capsule treatment of the solar system and other celestial objects, this book is recommended.

DIGITAL COMPUTER PRIMER, by E. M. McCormick. 214 pages, diagrams, 6 × 9 in. New York, McGraw-Hill Book Co., Inc., 1959. Price, $7.50.

The author, head of the Data Assessment Division of the Naval Ordnance Laboratory in California, has prepared this primer to explain automatic digital computers to the well-informed layman. In twelve chapters, he covers the design, operation and uses of these computers, with simple diagrams to accompany his explanations. He relates the processes to concepts known to the layman, in clearly expressed sentences.

CAUSALITY, by Mario Bunge. 380 pages, diagrams, 5½ × 8½ in. Cambridge, Mass. Harvard University Press, 1959. Price, $7.50.

In this volume the author analyzes the meaning of the causal principle and its place in modern science. It places special emphasis on causal determinism and critically examines the various doctrines of causation thus far produced. To estimate the weight of causation relative to other categories of lawful production, such as interaction and structural, statistical, dialectical and teleological determination, the author studies how these categories of determination actually work in the main departments of modern science.

THE ATLANTIC CABLE, by Bern Dibner. 96 pages, illustrations, 8 × 10¾ in. Norwalk, Conn., Burndy Library, 1959. Price, clothbound, $3.50; paperbound, $2.50.

The engineering achievement of a century ago, laying of the Atlantic Cable, is the subject of Bern Dibner's historical narrative. It is clearly written in a lucid style and abounds in real-life drama. Accompanied by many handsome illustrations and several maps, the book is a worthwhile addition to the library of the historian, scientist, engineer or the serious student. A bibliography of 60 books and pamphlets, published from 1857 to 1910 is included. The success of the transatlantic cable fired popular imagination as strongly as does the conquest of space today.

WATER FACTS FOR THE NATION'S FUTURE, by Walter B. Langbein and William G. Hoyt. 288 pages, diagrams, 6 × 9 in. New York, The Ronald Press Co., 1959. Price, $5.00.

It is expected that this nation's demand for water will increase possibly 70 per cent by 1975. This book analyzes the present hydrologic program and outlines the steps to be taken if that increased demand is to be met. Sponsored by The Conservation Foundation, the book should be read by water engineers as well as by city, state and regional adminis-

trators and legislators. The authors divide the book into two parts—one deals with the current programs for gathering data on water, the other discusses the effectiveness of the use of the data so gathered. The final chapter, "Better Data for Better Decisions," is a plea for making the best use of the data on hand.

PROCEEDINGS OF THE FOURTH CONFERENCE ON MAGNETISM AND MAGNETIC MATERIALS. 323 pages, illustrations, $7\frac{3}{4} \times 10\frac{1}{2}$ in. New York, McGraw-Hill Book Co., Inc., 1959. Price, $10.00.

Published as a supplement to Volume 30 of the *Journal of Applied Physics*, this book contains the 143 papers presented at a conference held in Philadelphia in November, 1958. Subjects covered include ferrites, computer components, micromagnetics, magnetic properties of metals and alloys, fine particles, amplifiers and microwave applications, resonance, metallurgical considerations, fundamental interactions, instrumentation, thin films, neutron diffraction and irradiation, garnets and other compounds, and anisotropy.

ELECTRONIC DIGITAL COMPUTERS, by Charles V. L. Smith. 443 pages, diagrams, 6×9 in. New York, McGraw-Hill Book Co., 1959. Price, $12.00.

This new book presents a comprehensive picture of the basic principles embodied in computing machines in use today and gives a thorough account of what electronic digital computers are and how they function. Although a good deal of the text is devoted to the physical structure of machines, it is not a treatise on digital computer engineering. The discussion of circuits and components provides the reader with a complete understanding of various ways in which the usual computer functions. The book contains material not only on up-to-date machines, but also information on the superspeed machines which will appear in the next few years.

PRINCIPLES OF CIRCUIT SYNTHESIS, by Ernest S. Kuh and Donald O. Pederson. 244 pages, diagrams, 6×9 in. New York, McGraw-Hill Book Co., 1959. Price, $8.50.

In this text, the topic of synthesis is introduced with a discussion of typical communication and control systems. Designed to acquaint the reader with the fundamental concepts of modern circuit synthesis, the author turns to discussing the approximation problem and the basic techniques of network realization. Illustrations and problems are chosen from practical system applications.

ELEMENTARY PRACTICAL HYDRAULICS OF FLOW IN PIPES, by C. T. B. Donkin. 202 pages, diagrams, $5\frac{1}{4} \times 8\frac{1}{4}$ in. New York, Oxford University Press, 1959. Price, $3.40.

This is a handy reference guide for the design, layout, installation and operation of the supply distribution and circulating systems of liquids. Some attention has been given to the economics of pipe system design. As a text book its scope is restricted and treated in a sufficiently elementary fashion to make it useful for draughtsmen and others who may not have studied the science of hydraulics to an advanced standard. At the same time it gives a very useful recapitulation of flow theory to those who have studied the subject.

ENGINEERING STATISTICS, by Albert H. Bowker and Gerald J. Lieberman. 585 pages, diagrams, 6×9 in. New York, Prentice-Hall, Inc., 1959. Price, $11.00.

Beginning with a clear introduction to the basic fundamentals of probability and distribution theory, this text thoroughly explains almost every major statistical technique. Carefully selected examples illustrate exactly how statistics are used to solve a wide variety of typical engineering problems. To help simplify an understanding of the analysis of variance, the authors distinguish clearly between the fixed, random, and mixed effects model. For the first time in any book, Operating Characteristic Curves appear for each of these models. A useful reference for both students and professionals in all areas of engineering and the physical sciences.

MODERN GEOMETRICAL OPTICS, by Max Herzberger. 504 pages, diagrams, 6×9 in. New York, Interscience Publishers, Inc., 1958. Price, $15.00.

Volume VIII in the Pure and Applied Mathematics series, this book on optics was

fourteen years in preparation. The author's aim is to develop a mathematical model of an optical system complex enough to encompass all the characteristics of the geometrical optical image, yet readily intelligible. The book is divided into ten parts, covering ray-tracing, precalculation of optical systems, general laws, concentric systems, rotation-symmetric systems, approximation theory for normal systems, third- and fifth-order image-error theory, interpolation theory of the optical image, optics in general media, and a four-part appendix. Higher mathematics is kept to a minimum, since the book is intended to be of use to lens designers; the necessary mathematical methods involved are found in the appendix. As an example of the simplified treatment throughout, the section on chromatic aberrations makes use of dispersion formulae that are linear functions of the indices. An eight-page bibliography includes references as far back as 1604 and up through 1957.

MATERIALS AND PROCESSES OF ELECTRON DEVICES, by Max Knoll assisted by B. Kazan. 484 pages, illustrations, plates, $6\frac{1}{2} \times 9\frac{1}{2}$ in. Berlin, Springer-Verlag, 1959. Price, DM 66.

The unusual interest in the subject of electron devices is evidenced by the fact that the dust jacket of this book is printed in four languages, although the text is entirely in English. Fourteen chapters cover the preparation and working of materials used in electron devices, finishing techniques such as degassing and getter procedures, and the production of solid state devices. An unusually detailed table of contents supplements the subject index. A bibliography of about 2300 references gives some indication of the thoroughness with which this text was prepared. In addition to the quantity of excellent black and white illustrations, the book contains four color plates showing stress patterns in glass-metal sealings and in a glass tube, leak detectors at different pressures, and the degassing of a graphite anode.

ANALYSIS OF LINEAR SYSTEMS, by David K. Cheng. 431 pages, diagrams, 6×9 in. Reading (Mass.), Addison-Wesley Publishing Co., Inc., 1959. Price, $8.50.

Designed as a text for advanced undergraduate or graduate level students in electrical or mechanical engineering, physics or applied mathematics (the preface states which chapters to use for these various courses), this book furnishes a thorough exposition of the techniques of formulating and solving equations in linear physical systems. Fourier series and the Fourier integral are covered, as are the Laplace transform method and the Laplace inversion integral. Illustrative examples and problems to be worked out follow each chapter. Answers to the problems are contained in a section at the back of the book.

PUBLICATIONS RECEIVED

THE CHALLENGE OF SCIENCE EDUCATION, edited by Joseph S. Roucek. 491 pages, 6 × 9 in. New York, Philosophical Library, 1959. Price, $10.00.

AN APPROACH TO ELECTRICAL SCIENCE, by Henry G. Booker. 826 pages, diagrams, 6 × 9 in. New York, McGraw-Hill Book Co., Inc., 1959. Price, $9.50.

MANUAL OF INSECT MORPHOLOGY, by E. Melville DuPorte. 224 pages, diagrams, 6 × 9 in. New York, Reinhold Publishing Corp., 1959. Price, $5.00.

HANDBOOK OF PRACTICAL ELECTROPLATING, by Thomas M. Rodgers. 333 pages, 5 × 7½ in. New York, The Macmillan Company, 1959. Price, $8.50.

MODEL RADIO-CONTROL, by Edward L. Safford, Jr. 192 pages, illustrations, 5½ × 8½ in. New York, Gernsback Library, Inc., 1959. Price, $2.65 (paper).

INTRODUCTION TO ELECTRONIC ANALOG COMPUTERS, by John N. Warfield. 175 pages, diagrams, 6 × 9 in. New York, Prentice-Hall, Inc., 1959. Price, $6.00.

FUNDAMENTALS OF ELECTRICAL ENGINEERING, by Fred H. Pumphrey. Second edition, 534 pages, illustrations, 6 × 9 in. New York, Prentice-Hall, Inc., 1959. Price, $8.50.

OPTICAL MINERALOGY, by Paul F. Kerr. Third edition, 442 pages, illustrations, 6 × 9 in. New York, McGraw-Hill Book Co., Inc., 1959. Price, $8.50.

A PRIMER OF PROGRAMMING FOR DIGITAL COMPUTERS, by Marshal H. Wrubel. 230 pages, illustrations, 6 × 9 in. New York, McGraw-Hill Book Co., Inc., 1959. Price, $7.50.

SOME PROBLEMS IN CHEMICAL KINETICS AND REACTIVITY. Volume 2, by N. N. Semenov, translated from the Russian by Michel Boudart. 330 pages, diagrams, 6 × 9 in. Princeton, Princeton University Press, 1959. Price, $4.50 (paper).

ORGANIC CHEMISTRY, by Astle Shelton. Second edition, 771 pages, diagrams, 6 × 9¼ in. New York, Harper & Brothers, 1959. Price, $7.50.

PRINCIPLES OF DIRECT-CURRENT MACHINES, by Alexander S. Langsdorf. Sixth edition, 363 pages, illustrations, 6 × 9 in. New York, McGraw-Hill Book Co., Inc., 1959. Price, $8.50.

TECHNICAL CALCULUS AND ANALYSIS, by Harold S. Rice and Raymond M. Knight. 460 pages, diagrams, 6 × 9 in. New York, McGraw-Hill Book Co., Inc., 1959. Price, $6.75.

FUNDAMENTALS OF FRESHMAN MATHMEMATICS, by C. B. Allendoerfer and C. O. Oakley. 475 pages, diagrams, 6 × 9 in. New York, McGraw-Hill Book Co., Inc., 1959. Price, $6.50.

ELECTRIC MOTORS & GENERATORS, by E. T. G. Emery, F. Harrabin, B. C. Lee, J. H. R. Nixon and E. T. A. Rapson. 384 pages, illustrations, 5½ × 8½ in. New York, Philosophical Library, 1959. Price, $12.00.

AUTOMATION AND SOCIETY, edited by Howard Boone Jacobson and Joseph S. Roucek. 553 pages, 5½ × 8½ in. New York, Philosophical Library, 1959. Price, $10.00.

TERMS USED IN ARCHAEOLOGY, by Christopher Trent. 62 pages, 4½ × 7½ in. New York, Philosophical Library, 1959. Price, $2.75.

ELEMENTARY STATISTICS WITH APPLICATIONS IN MEDICINE AND THE BIOLOGICAL SCIENCES, by Frederick E. Croxton. Unabridged and corrected republication of the work first published under the title "Elementary Statistics with Applications in Medicine." 376 pages, diagrams, 5½ × 8 in. New York, Dover Publications, 1959. Price, $1.95 (paper).

PILLOW PROBLEMS AND A TANGLED TALE, by Lewis Carroll. Unabridged and unaltered republication of the following two books by Lewis Carroll: "Pillow Problems" and "A Tangled Tale." 261 pages, diagrams, 5½ × 8 in. New York, Dover Publications, 1958. Price, $1.50 (paper).

SYMBOLIC LOGIC AND THE GAME OF LOGIC, by Lewis Carroll. Unabridged and unaltered republication of the following two books by Lewis Carroll: "Symbolic Logic Part I, Elemen-

tary" and "The Game of Logic." 295 pages, diagrams, 5¼ × 8 in. New York, Dover Publications, 1958. Price, $1.50 (paper).

FRUIT KEY AND TWIG KEY, by William M. Harlow. Unabridged and unaltered republication of the first edition of "Fruit Key to Northeastern Trees" and the fourth revised edition of "Twig Key to the Deciduous Woody Plants of Eastern North America." 106 pages, illustrations, 5½ × 8½ in. New York, Dover Publications, 1959. Price, $1.25 (paper).

THE MEASUREMENT OF POWER SPECTRA, by R. B. Blackman and J. W. Tukey. Unabridged and corrected republication of Part I and Part II of "The Measurement of Power Spectra from the Point of View of Communications Engineering." 190 pages, diagrams, 5½ × 8 in. New York, Dover Publications, 1959. Price, $1.95 (paper).

ADVANCED CALCULUS, by Edwin Bidwell Wilson. Unabridged and unaltered republication of the first edition. 566 pages, 5½ × 8½ in. New York, Dover Publications, 1959. Price, $2.45 (paper).

THE DYNAMICS OF PARTICLES, by Arthur Gordon Webster. Unabridged and unaltered republication of the second edition. 588 pages, diagrams, 5½ × 8 in. New York, Dover Publications, 1959. Price, $2.35 (paper).

COMPUTATIONAL METHODS OF LINEAR ALGEBRA, by V. N. Faddeeva, translated from the Russian by Curtis D. Benster. 252 pages, 5½ × 8 in. New York, Dover Publications, 1959. Price, $1.95 (paper).

THEORY OF FUNCTIONALS AND OF INTEGRAL AND INTEGRO-DIFFERENTIAL EQUATIONS, by Vito Volterra. Unabridged republication of the first English translation, with preface and biography and bibliography added. 226 pages, 5½ × 8 in. New York, Dover Publications, 1959. Price, $1.75 (paper).

MESON PHYSICS, by Robert E. Marshak. Unabridged and unaltered republication of the first edition. 378 pages, diagrams, 5½ × 8 in. New York, Dover Publications, 1958. Price, $1.95 (paper).

APPLIED DESCRIPTIVE GEOMETRY, by Frank M. Warner and Matthew McNeary. Fifth edition, 243 pages, diagrams, 6½ × 9½ in. New York, McGraw-Hill Book Co., Inc., 1959. Price, $5.25.

MACHINE DESIGN PROBLEMS, by Donald J. Myatt. 186 pages, diagrams, 8½ × 11 in. New York, McGraw-Hill Book Co., Inc., 1959. Price, $5.50 (paper).

MINERALOGY, by Edward Henry Kraus, Walter Fred Hunt and Lewis Stephen Ramsdell. Fifth edition, 686 pages, illustrations, 6 × 9 in. New York, McGraw-Hill Book Co., Inc., 1959. Price, $9.00.

PARTIAL DIFFERENTIAL EQUATIONS OF MATHEMATICAL PHYSICS, by Harry Bateman. Reprint of first edition, 522 pages, 6 × 9¼ in. New York, Cambridge University Press, 1959. Price, $4.95 (paper).

CURRENT TOPICS

Miniaturized Artificial Larynx under Development.—A new artificial larynx for persons who have lost their voices through surgical removal (laryngectomy) or paralysis of their vocal cords was described by three Bell Telephone Laboratories scientists at a recent meeting of the Acoustical Society of America, in Ottawa, Canada. Still in the experimental stage, it is the result of a considerable background of research in an interdisciplinary field of science known as psychoacoustics.

Great impetus to development of the device was given by some of the nation's foremost surgeons connected with the National Hospital for Speech Disorders. They felt that, what with the great advances in electronics brought about by the transistor, specialists in acoustics research could devise a far better artificial larynx than any presently available.

With a minimum of difficulty and training, laryngectomees can use the new electronic larynx to speak conversationally. It is especially effective when conversing over the telephone.

By means of a finger-operated combination push-to-talk switch and inflection control, the user can easily control the pitch of his artificial voice, thus giving his speech a natural sounding quality previously unobtainable.

The underlying principle of the new artificial larynx is a vibrating driver (transducer) held against the throat. Completely self-contained and cylindrically shaped, it measures only $1\frac{3}{4}$ inches in diameter by $3\frac{1}{4}$ inches long—thus acceding to plaints of laryngectomized people for an unobtrusive device. Included in this one small package is a modified telephone receiver serving as the throat vibrator, a highly-efficient transistorized pulse generator with pitch contol, and a battery power supply. In order to miniaturize the new artificial larynx, experimental units were built using modular techniques. However, by employing printed-circuit techniques it is anticipated that an even more compact unit can be built.

To use the unit, the laryngectomized person presses the vibrator against his throat. Switching on the pulse generator with his finger, he transforms vibrations transmitted into his throat cavities into speech sounds by normal use of the articulatory mechanisms—that is, throat cavity or pharynx, tongue, mouth, teeth, and lips—in his vocal tract.

Output speech volume obtained with the artificial larynx is equal to that of a person speaking at a normal conversational level, though the sound is a bit buzzy and mechanical. Nevertheless, the frequency spectra of vowel sounds show that the frequency range transmitted into the person's throat is sufficient for satisfactory production of such sounds. And while intelligibility tests give results lower than those of normal speech, they are superior to those of any other artificial larynx. Users of the new artificial larynx can achieve a sentence intelligibility of 97 per cent or more, depending on their experience.

Because the artificial larynx requires an economical, self-contained power source, circuit parameters had to be adjusted to yield maximum acoustic output with a minimum of current drain. Accordingly, two

V.2
195

transistors are used in a relaxation oscillator whose frequency is controlled by a variable resistance, and whose pulse width is determined by a feedback network. The output is a negative pulse which occurs at a frequency of about 100 cycles per second. This repetition frequency may be varied from about 100 to 200 cycles by a rheostat which the user operates by pressure on the push-to-talk switch—or inflection control—while speaking, thus changing the pitch of his voice. For use by women talkers, the frequency range is adjusted to 200 to 400 cycles, to correspond with the normal range of pitch of a woman's voice.

A third transistor acts as a single-ended power output stage that amplifies the pulses applied to it from the relaxation oscillator. A diode serves to isolate the multivibrator from the power amplifier input impedance during the period between pulses, and is necessary for stable operation. Because a large pulse is required for sufficient acoustic power output at low frequencies, the relaxation oscillator drive circuit has heavy current requirements.

Two 5.2-volt mercury cells in series provide the power necessary to operate the artificial larynx continuously for a period of approximately 12 hours. These batteries have a 250-milliampere hour rating with a maximum permissible current drain of 25 milliamperes. With push-to-talk operation such as the laryngectomized patient requires, 12 hours of continuous operation should be equivalent to several days or even weeks of normal talking.

An alternative to using the self-contained mercury cells for powering the artificial larynx is a small a-c power supply which can be fed from a normal wall outlet at home or in the office. When the artificial larynx is plugged into the power supply, its batteries are disconnected from the circuit.

New Character Generator.—A new, high speed method of generating alpha-numeric characters for electronic display purposes has been developed by Philco Corporation's Government and Industrial Division. This new Philco character generator can be used for continuous display of tabular information and/or insertion of written data into pictorial-type displays by means of time-sharing techniques.

Electronic character generation can be used in a wide variety of situations. For example, the Philco generator is especially suited for air-traffic control operations and can be adapted for high speed computer read-out functions.

This new centrally located, on-command system developed by Philco engineers produces bright, highly legible characters from a minimum amount of equipment. The techniques applied offer improvements over character generators now on the market in the infinite selection of characters, faster writing speed, reliability and economy. Characters are written by the electron beam in a cathode ray tube just as they would be lettered on a sheet of paper. However, no special character writing tubes are needed.

A typical, desirable application of the character generator uses a dual deflection (magnetic and electrostatic) cathode ray tube. Major positioning of inputs is applied to the magnetic deflection yoke, while the voltage waveform applied to the electrostatic deflection plates sweeps out the desired character. Use of a focused writing beam results in sharp, well-defined and easily read characters.

The basic character generator module provides a selection of up to

64 characters of any shape or size, including geometric descriptive symbols. Also, one character generator can drive any number of displays simultaneously.

Writing speeds can be varied over a wide range up to 50,000 characters per second in the present design. Selection of characters can be easily changed by inserting a new character mask. Characters to be written may be specified by digital inputs if desired, permitting remote location of the character generator from the selection and display equipment.

Random time selection of characters permits easy interlace of characters into fixed, recurrent, time-sharing display sequences.

Philco's character generating technique is very simple and readily adaptable to circuit reduction requiring very few sub-units for the over-all system. An entire system for generating 64 characters can be packaged in a single standard equipment rack.

Super-cooled Infrared Detection System.—A tiny cooling device which super-chills infrared detection equipment to 60 K. (−350 F.) by a new refrigerating technique may prove to be a significant scientific achievement in support of U. S. military and civil aviation. Since all matter generates infrared energy, every object, man-made or natural, is subject to infrared detection.

The 8-oz. min-IR-cooler is the result of a pioneering two-year research project into extreme low temperature equipment conducted by Dr. Howard O. McMahon and William E. Gifford at Arthur D. Little, Inc., Cambridge research and engineering company. Development of the system for infrared applications is being conducted by ADL and Hamilton Standard, Division of United Aircraft Corp., Windsor Locks, Conn. The latter holds an exclusive license from ADL to manufacture the min-IR-cooler for infrared applications only.

Ideal for airborne operation in missiles or airplanes, the present closed-cycle detection system is designed to weigh less than 20 pounds. Further development is expected to reduce this weight to less than ten pounds.

In operation, helium gas expands from 300 psi. in a ¼-in.-diameter cylinder, 2 in. long. A tiny plastic piston is the only moving part of the unit below room temperature. The cold end of the tube refrigerates an infrared cell to 60° K.

The designers pointed out that cooling infrared detectors to extremely low temperatures increases their sensitivity and makes them responsive to a wider range of IR wave lengths. This makes it possible to detect small temperature differences between the "target" and its surroundings.

An airborne IR detector with a min-IR-cooler, for instance, would be able to detect the presence of an aircraft or missile from a great distance.

The need for a miniature, reliable refrigerator for extremely low temperatures has been recognized for several years. No completely satisfactory system has been devised before this.

Convertiplane Described.—A new aircraft that hovers like a helicopter, but flies as far and as fast as a propeller-driven plane, was described recently by Claude E. Leibensberger, project flight test engineer for the Bell Helicopter Corporation, Fort Worth, Texas. He outlined the new craft's abilities to the Semi-Annual Meeting of The American Society of Mechanical Engineers in St. Louis.

The vehicle, called a "converti-

plane," has propellers that can be used vertically, horizontally or at any position in between. For tight-squeeze takeoffs or reconnaissance, the pilot aims the propellers straight up. For high-speed, long-distance flight, he tilts them forward. The plane will be able to range over 3000 miles and, because it could land in downtown city areas, may be faster, timewise, than any long-range planes, except jets. It is expected to have a cruise speed close to 350 miles per hour. The plane can be used for observation, reconnaissance and rescue missions and larger versions for utility, assault and transportation.

In addition, if the plane is used as a "straight" aircraft—with propellers forward—tests indicate that it will be able to take off within 500 to 600 ft. with a 150 per cent increase in cargo over the helicopter version, said Mr. Leibensberger. The additional power built in for helicopter use can push the plane up in a shorter time than a conventional aircraft could rise. Expected helicopter cargo versions will carry two to three tons and the short take-off and landing versions will be able to handle four to six tons.

The pilot makes the propellers rotate upward or forward by moving an electro-mechanical conversion switch. He can stop the operation at any point. The whole manoeuver can be completed in twelve seconds.

The pilot also has a gear shift so that he can use high engine speed for helicopter and initial plane configuration and lower engine speed to increase propeller efficiency and long-range economy once the props are in forward position.

A moderate-powered prototype of the plane has been tested successfully and the future for the dual purpose plane looks bright, said Mr. Leibensberger.

Fuel Cell Developed.—A device that produces electricity directly from gaseous fuel has been developed in a new, improved form by General Electric scientists. For more than a century, scientists have been trying to develop a successful "fuel cell," as such a device is called, because it offers the possibility of generating electricity much more efficiently than by conventional methods and because it is a compact, light-weight power source. The fuel cell generates electricity by means of a chemical reaction. In the present version, hydrogen and oxygen produce an electric current, with water as a by-product.

The model, developed by Dr. W. Thomas Grubb and Dr. Leonard W. Niedrach, consists of a round plastic disk about ½ in. thick and 3 in. in diameter. Its hollow interior is divided into two chambers by a special plastic membrane, which has an electrode in contact with each of its sides. Hydrogen is fed into one chamber, and oxygen into the other (or oxygen in the air can be used). At one electrode, the hydrogen molecules break up into electrons and positively charged hydrogen. The electrons travel through an external circuit to the other electrode, thus creating an electric current. The positively charged hydrogen moves through the membrane to the other electrode, where it combines with oxygen and the electrons from the external circuit to form water.

The new device operates efficiently at room temperature and normal atmospheric pressure. Thermal efficiencies over 60 per cent have been obtained.

The broad principles of this device are common to all fuel cells, but the type described above is novel in employing an *ion exchange membrane* as an electrolyte. This has an advan-

tage over aqueous electrolytes, in that the electrolytically conducting ions are "locked in," and cannot be leached from the cell. These membranes also reject water when they reach saturation, and, therefore, no difficulty is experienced from dilution effects. These features combine to simplify the operation of the cell, because they provide built-in controls for maintaining the electrolyte concentration and the water balance.

In addition, the electrolyte may be *acidic*, consisting of a cation exchange resin in its hydrogen form. The use of an acidic electrolyte makes it possible to use gases containing carbon dioxide, for example, air.

In cells using caustic electrolytes, such as potassium hydroxide, the hydroxyl ion is the charge carrier, and the water produced by the net cell reaction forms at the fuel electrode. In the present device, the hydrogen ion is the charge carrier, and the water forms at the oxidant electrode. This is advantageous when the cell is operated on air, since the water can be disposed of by controlled evaporation.

Among the other features that should be noted are the following: the solid polymer electrolyte makes possible very compact cell structure, operation on static heads of gas is possible, no moving parts are involved, and close tolerances are not involved in the construction of the cell.

A number of cells have been operating continuously for from 75 to 100 days, and these tests have not yet been concluded. One cell has operated for more than 100 days, and is still running. Two cells have been operated with their cathodes exposed to the air; one of them is still running after more than 75 days. Portions of these tests have been conducted at current densities of 30 ma/cm², although most of the operation has been at lower current densities.

The cells have been operated mainly on commercially available hydrogen and oxygen.

Because of the thinness of the cell itself and the small size of the gas chambers, the performance of the cell in terms of net power per unit weight (or per unit volume) is attractive. Power densities of the order of one to two kilowatts per cubic foot have been calculated on the basis of present data.

The fuel cell may find both civilian and military uses, even though it produces low-voltage direct current, since its special characteristics may well fit it for a variety of uses. It is likely that the present cell will be useful for specialty applications, rather than for the production of large quantities of power for general distribution. Military and space vehicle applications might make use of the fuel cell's high reliability, simplicity, portability, light weight, and small volume.

Flexible Inorganic Films Provide High-Temperature Electrical Insulation.—Many potentially important applications of electrical circuitry at high temperatures are presently hindered by the lack of high quality flexible wire insulation. Recent discoveries at Bell Telephone Laboratories indicate that fluoride coatings can be formed on copper, aluminum and other metal wires which will provide exceptionally high insulation value at elevated temperatures, while retaining flexibility and freedom from porosity.

According to a paper by Drs. Steward S. Flaschen and Paul D. Garn, which was presented to a recent meeting of the American Ceramic Society, the insulating coatings are formed directly on freshly cleaned copper or aluminum by exposing them

to oxidizing carriers of fluorine such as hydrogen fluoride or elemental fluorine at temperatures from 300 C. to 600 C. The thickness of the resulting copper fluoride and aluminum fluoride films depends on the temperature at which they are formed, the concentration of fluorine and the time of exposure. Aluminum forms a fluoride film one micron thick in a few minutes at 550 C. These films remain adherent when bent repeatedly at 90° angles.

Electrical insulation values are very high for both copper and aluminum films, being in the order 10^{10} and 10^{11} ohms at room temperature for films in the order of 1 to 2 microns thick, between probe electrodes $\frac{1}{4}$ in. in diameter. The films retain their excellent insulating properties at high temperatures. For example, aluminum fluoride films exhibit resistances of about 7×10^8 ohms at temperatures as high as 500 C.

The aluminum fluoride films show excellent resistance to oxidation even above 600 C. They also show no tendency to hydrate or dissolve on exposure to high humidity. The insulation does not break down even at 450 volts at 500 C.

The best organic insulating coatings cannot be used continuously above 300 C. Inorganic insulators are generally porous and non-flexible, although some may be used as high as 830 C. Insulating fluoride coatings should be satisfactory almost up to the melting point of the conductor.

This new approach to insulating could provide the answer to some of the problems in missile re-entry guidance systems, as well as other more earth-bound high temperature electrical applications. The technique is not limited to copper and aluminum according to its inventors, but can be used to form thin flexible films on other metals as well.

Small Ultrasonic Welder.—A new instrument-type ultrasonic welder designed to make welds in small parts and delicate assemblies has been developed by Aeroprojects of West Chester, Pa. This unit, with a power capacity of 100 watts, is designated SONOWELD Model W-100-TSL-58-6. The small, new welder offers several important advantages to manufacturers in electronic and instrument fields. Because ultrasonic welding is accomplished without melting or fusion and since no electrical current passes through the parts being joined, contamination of surrounding areas by sputter, arcing or spatter is eliminated.

Many difficult-to-join metal combinations are handled in stride by the SONOWELD unit. For example, superior ohmic contacts are routinely made between such semi-conductors as silicon or germanium and aluminum or gold wire. Electric match and other fine bridge wire assemblies using high resistance wire in the thickness range of 0.003 to less than 0.001 in. are welded by SONOWELD with a high degree of reproducibility. A large number of other dissimilar metal combinations are also joined successfully.

The latest and smallest ultrasonic welding unit further expands Aeroprojects' line of SONOWELD spot-type welders which includes large production-type machines of 2000- and 4000-watt capacity plus a 300-watt capacity model.

Upper Atmosphere Balloon.—Man may soon learn the exact density of the atmosphere 300 miles above the earth by means of information relayed back from a unique balloon ejected

from a two-stage rocket. During the past year, the Cambridge (Mass.) research and engineering company of Arthur D. Little, Inc., has overcome complex engineering problems involved in designing the device.

As a research project for the Geophysics Research Directorate of the Air Force Cambridge Research Center in Bedford, Mass., the consulting company developed an inflated sphere with instrumentation and a telemetry system. Purpose of the experiment is to measure density drag up to 300 miles into previously uncharted areas.

First stage of the flight will be furnished by an Aerobee research rocket. The second stage, solid propellant Sparrow rocket, will soar up to 300 miles. About 60 miles up, well above the burnout point of the Sparrow, a timed signal will trigger the rocket's mid-section, causing it to fold at a 90-degree angle, thus reducing the spinning motion. Another signal will eject an 18-lb. package through a door in the rocket's shell. Upon release, the package will blossom in a 9-ft. mylar plastic sphere, reinforced with dacron threads. Simplest method of inflation, the engineers discovered, was to allow ethyl alchol to evaporate and inflate the balloon at low pressure.

A small capsule containing freon gas at a higher pressure will discharge into a hollow plastic strut inside the balloon. At mid-point of this strut will be an aluminum can containing instruments to record and transmit data to earth. At the same instant a radio transmitter within the can will begin to send to a ground station. Information transmitted will note the amount of drag induced by the rarefied gases in the upper atmosphere acting upon the outer surface of the balloon. As the balloon is freed from the rocket it closely follows the trajectory of the projectile up to the zenith of 300 miles.

Arthur D. Little, Inc., engineers have calculated that this free flight of the balloon will take between 7 and 8 minutes. During this period valuable information will be continuously recorded on instruments far below.

The special accelerometer, heart of the instrument package, was designed by the University of Michigan and further modified by the Air Force Cambridge Research Center to measure up to 100,000th of a g. Originally it was encased in a 7-in. aluminum sphere and provided density measurements to about 80 miles up.

Shock-proof Radio Beacon.—A radio beacon designed to withstand shocks of over 5000 g applied in any direction has been developed by the General Electric Company. Easily capable of surviving a fall from the top of the Empire State Building, the beacon was developed for use in recovering data capsules after freefalling 25,000 ft. from a re-entering nose cone.

Signals from the beacon have been picked up at over 100 miles. With a peak output of 15 watts, the beacon operates in the UHF frequency band, but can be modified to other frequencies. Weighing only 18 oz., the beacon is powered by mercury cells. A 2-lb. power supply lasts for 1½ days of continuous operation.

The beacon has been used by G. E.'s Missile and Space Vehicle Department in successfully locating data capsules used in the nose cones of Thor and Atlas missiles. Other anticipated uses are for homing devices, renewal buoys, and distress aids.

The circular form of the present model can be modified to other desired configurations.

Electronic Microfilm Printer for Social Security Data.—The Social Security Administration has put a new electronic microfilm printer into op-

eration at its Bureau of Old-Age and Survivors Insurance headquarters to speed updating of earnings records of the 130 million people to whom social security cards have been issued. The machine does the job six times faster than was possible previously.

The special microfilm printer was designed and built to Social Security's specifications by the Special Engineering Products Division of International Business Machines Corporation. It converts data coded on magnetic tape directly to printed, readable records on 16mm microfilm. The machine can print at a rate of more than 3,000 lines—each containing 120 characters—a minute. It could print the entire text of an average length novel in 2 minutes.

Joseph L. Fay, Assistant Director in charge of Accounting Operations, Bureau of Old-Age and Survivors Insurance, estimates that the printer will save the Administration $200,000 a year.

The Bureau is presently using the printer to keep its National Employee Index up-to-date. The Index contains information on the 130 million social security card holders, and is referred to, for example, whenever a worker's survivors apply for benefits but do not known his social security number, or when a worker loses his card and applies for a duplicate. Three million duplicate cards are issued yearly.

In printed form, the file took up a city block of floor space and, at the time the microfilm printer was installed by IBM, was growing at the rate of 3 million names, requiring 6,000 square feet of additional floor space, each year.

Condensed on only 2,500 reels of microfilm, the Index and the readers used by clerks in referring to the file, require only 16,000 feet of floor space. In addition, the film rolls have enough blank space on them to accommodate earnings to be added during the next 20 years.

The microfilm printer was designed primarily for making microfilm records of the detailed listings of earnings posted to the social security accounts of the nation's workers every three months. Before the development of the microfilm printer, the magnetic tape listings had to be translated to paper and then microfilmed in a separate step. Subsequently the paper was thrown away. Now, by recording directly from tape to microfilm, the IBM printer eliminates the need for producing the paper listings.

The microfilm printer can handle the entire printing task required in the earnings listing application—1,200,000 lines a day—in less than an eight hour period.

The equipment is composed of four units: two magnetic tape units, a printer control unit, and a microfilm output printer system.

Coded information read by the magnetic tape units is extracted by the printer control unit which translates it to alphabetic and numerical characters positioned on the face of a cathode ray tube. This tube, similar to a TV picture tube, then flashes the characters to the microfilm output printer where the microfilming operation takes place.

Installation of the equipment is the latest step taken by the Social Security Administration in automating its operations. In March 1956, the first of two IBM 705 computers to be delivered to Baltimore headquarters of the Bureau of Old-Age and Survivors Insurance was installed. This 705's principal applications were transferring workers' earnings records—contained in 120 million IBM punched cards—to magnetic tape, and tracking down correct social security numbers

for the nine million earnings items incorrectly reported each year.

At present the two 705's are handling a variety of tasks, including the computation of benefits, updating of earnings records every three months and compiling social security statistical data.

Machine Reads Typewritten Pages.—The United States Air Force Air Research and Development Command has announced the successful development of a new information machine which reads typewritten pages and translates them into electrical signals at the rate of 200 characters per second. The new machine, the first of its kind, was developed by the Intelligent Machines Research Division of Farrington Manufacturing Company, Arlington, Virginia, and called the Print Reader MX-2021.

As it scans a line at a time, it reads the information one character at a time, and upon identification, using stored electronic logic, it instantaneously converts the information into electrical signals at the rate of 200 characters per second. It thereby provides the Air Force with information in a form which avoids all subsequent manual handling in communications, indexing, abstracting, and other information-processing problems necessary to the Air Force and other government agencies. The Air Force interest in the Print Reader centers upon a never ending maintenance of millions of pages of information which must be processed before being used. All technical information must be indexed and abstracted so scientists will know the information exists and locate the information when they need it.

While machines already have been used to translate, index, abstract and retrieve information, the original data have always been entered by keyboard operators manually. Now the Print Reader performs this heretofore tedious operation almost instantaneously and accurately, and relieves manpower for more important tasks required in information data-handling within the Air Force. With only thirty of these machines, it is possible automatically to read and process information at a rate of ten million pages a year.

The Air Force's reading machine at the present can read only one type style; however, continuing research promises to achieve reading all type styles in the very near future.

The USAF Print Reader, MX-2021, uses a scanner to produce TV-like video impulses from the typewritten characters on a page, and then analyzes the video pattern for various identification clues. Unlike most other electronic machines, this electronic reading machine can test for identification clues, continuously and simultaneously. This enables the Print Reader to make a very sophisticated analysis in identifying the pattern.

While processing characters at a rate of 200 per second, the reader translates the information in electrical signals for further machine processing. The output can be punched paper tape, punched cards, magnetic tape or fed directly into electronic computers.

Machines of this nature aid Air Force scientists in areas of research in pattern-recognition devices, self-organizing systems, cybernetics, computation and automatic programming, retrieval of information from exceedingly large files, and control of large information complexes.